# Robust Exclusion and Market Division through Loyalty Discounts

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## Abstract

We show that loyalty discounts create an externality among buyers because each buyer who signs a loyalty discount contract softens competition and raises prices for all buyers. This externality can enable an incumbent to use loyalty discounts to effectively divide the market with its rival and raise prices. If loyalty discounts also include a buyer commitment to buy from the incumbent, then loyalty discounts can also deter entry under conditions in which ordinary exclusive dealing cannot. With or without buyer commitment, loyalty discounts will increase profits while reducing consumer welfare and total welfare as long as enough buyers exist and the entrant does not have too large a cost advantage. These propositions are true even if the entrant is more efficient and the loyalty discounts are above cost and cover less than half the market. We also prove that these propositions hold without assuming economies of scale, downstream competition, buyer switching costs, financial constraints, limits on rival expandability, or any intra-product bundle of contestable and incontestable demand.

# 1 Introduction

In a loyalty discount contract, a seller commits to charge loyal buyers (those who buy all or a high percentage of the product from that seller) less than other buyers.[1] Prior analysis of loyalty discounts has focused on whether they should be treated like exclusive dealing (because buyers only obtain a discount if she buys little or no product from competitors) or like predatory pricing (because the seller is offering buyers lower prices through the discount). Part of the disagreement reflects differing assumptions about whether loyalty discounts involve buyer commitments similar to those in ordinary exclusive dealing or instead leave the buyer free to buy elsewhere if a rival offers a lower price. Both types of loyalty discounts are possible and present in actual markets, and it turns out that their analysis differs in certain ways.[2] To clarify the analysis, we model loyalty discounts with buyer commitment separately from loyalty discounts without buyer commitment.

First, we address loyalty discounts that a seller gives in exchange for a buyer commitment to remain loyal by not purchasing from the seller's rival. This analysis differs from that for ordinary exclusive dealing because loyalty discounts add a *seller* commitment to charge loyal buyers less than disloyal buyers. We show that this important feature of loyalty discounts creates distinctive anticompetitive effects that make loyalty discounts with buyer commitment especially effective at excluding rivals.

Second, we model loyalty discounts without buyer commitment, where buyers receive the

[1]This discount could be either a fixed dollar discount or a percentage discount. The discount could leave the actual price unspecified ex ante; the seller commits only that the price for loyal buyers will be less than the price for other buyers. Some assert that actual loyalty discounts do not involve any seller commitment to maintain a loyalty discount. See, e.g., Crane, pp. 286-288 (2013); Lambert (2012). In fact, such seller commitments are common in the health care industry, where dominant suppliers typically contract with hospitals through Group Purchasing Organizations (GPOs). In the typical contract, the supplier agrees with the GPO to offer contracts to GPO members with price tiers based on whether the hospital is loyal (buys a high percentage) from the supplier, with hospitals who agree to disloyal tiers getting a nominal "discount" from supplier-set list prices and those who agree to loyal tiers guaranteed a higher discount. See Elhauge (2002). These supplier-GPO contracts thus commit the supplier to charging loyal customers less than disloyal ones, even though both prices can be moved in tandem by changing the list price.

[2]Some assert that loyalty discounts generally do not involve any buyer commitments, see Crane, pp. 286-289 (2013), but in fact they often do. See Elhauge, pp. 2-8 (2002); see, e.g., Natchitoches Parish Hosp. Serv. Dist. v. Tyco Int'l, Ltd., 247 F.R.D. 253, 259 (D.Mass.2008) ("Tyco required purchasers to commit to buying a specific percentage of all of their sharps containers needs from Tyco in order to get the best pricing"); Natchitoches Parish Hosp. Serv. Dist. v. Tyco Int'l, Ltd.., 2009 WL 4061631, at *5-6 (D. Mass. Nov. 20, 2009) (distinguishing loyalty discounts with buyer commitments from those without them).

loyalty discount for buying a specified share from the seller but remain free to buy elsewhere if a rival offers a lower price. Many have analogized these loyalty discounts to predatory pricing.[3] Our model, however, highlights three crucial differences from ordinary predatory pricing: (1) in loyalty discounts, prices are conditioned on buying a certain share from the seller; (2) loyalty discounts need not involve any true discount from but-for prices because they merely set a difference between the prices charged to loyal and disloyal buyers that could just as easily involve raising the disloyal price above but-for levels; and (3) loyalty discounts involve a seller commitment to charge loyal buyers less than disloyal buyers. We show that these three features create distinctive anticompetitive effects that can make loyalty discounts more akin to market division than to predatory pricing.

The proper antitrust treatment of loyalty discounts has been a contentious issue. Some courts have held that loyalty discounts cannot be anticompetitive unless they are below cost, while other courts have rejected that proposition or held it applies only when price is the clearly predominant mechanism of exclusion.[4] In 2004, the Solicitor General advised the U.S. Supreme Court to avoid taking a case to resolve this legal conflict, in part because economic analysis on the topic was unresolved. Since then, the Supreme Court has continued not to intervene, while economic analysis on loyalty discounts has remained divided.[5]

For loyalty discounts *with* buyer commitment, we prove that, unlike ordinary exclusive dealing, they can have anticompetitive effects even if sellers lack economies of scale, buyers are final consumers, buyers can coordinate on their preferred equilibrium, and the contracts cover a minority of the market. Further, whereas for ordinary exclusive dealing a possible equilibrium involves all buyers rejecting anticompetitive exclusive dealing, we prove that universal buyer rejection is not a possible equilibrium if the entrant's cost advantage is not

[3]See, .e.g., Hovenkamp (2005); Lambert (2005); Hovenkamp (2006).

[4]Compare, e.g., Concord Boat v. Brunswick Corp., 207 F.3d 1039, 1061-62 (8th Cir. 2000) (must be below cost), with LePage's v. 3M, 324 F.3d 141, 147-52 (3d Cir. 2003) (en banc) (need not be); ZF Meritor, LLC v. Eaton Corp., 696 F.3d 254, 269, 275 277 (3d Cir. 2012) (loyalty discounts need be below cost only if "price is the clearly predominant mechanism of exclusion.").

[5]For articles arguing that, like predatory pricing, loyalty discounts presumptively lower prices and cannot harm consumer welfare in the long run unless they are below cost, see Hovenkamp (2005); Lambert (2005); Hovenkamp (2006). For scholarship arguing that loyalty discounts can create anticompetitive effects similar to exclusive dealing, see Tom, et al., pp. 615, 623-24, 627 (2000); Elhauge, pp. 284-92 (2003); Spector, pp. 99-101 (2005); Whinston, pp. 144-47, 166-67 (2006); Kaplow & Shapiro, pp. 1203 n.98 & 106 n.207 (2007); Elhauge, pp. 406-412 (2008).

too large and there are enough buyers. In any equilibrium, enough buyers accept the loyalty discount to anticompetitively increase prices and reduce total welfare, and there always exists a possible equilibrium where all buyers accept, completely foreclosing a more efficient rival. None of these results depends on loyalty discounts being below cost; instead, the effect of loyalty discounts is to increase prices even further above cost.

What drives the difference in effects is that the incumbent commitment to maintain a loyalty discount softens competition for free buyers. The loyalty discount reduces the incumbent's incentive to compete for free buyers because lowering the price to free buyers requires lowering the price to captive buyers. This, in turn, reduces the entrant's incentive to compete for free buyers with aggressive pricing. This increases prices to free buyers, which inflates prices to captive buyers because their price is based on the loyalty discount from free buyer prices. Prices are elevated above competitive levels to all buyers, reducing consumer and total welfare.

This raises the question how can a commitment to higher post-entry prices deter entry? Not surprisingly, an externality drives this effect. For each additional buyer that agrees to a loyalty discount with buyer commitment, competition becomes less aggressive, raising prices for all buyers. The incumbent supplier only needs to compensate each buyer for her losses from these higher prices, but she gains from the higher prices affecting all buyers. Thus, the incumbent's gain from signing an additional buyer can exceed this buyer's loss while still falling short of the losses this creates for all buyers. This enables the incumbent to profitably induce buyers to agree to loyalty discounts with buyer commitment that reduce overall welfare and potentially exclude an efficient entrant entirely.

While we prove these results generally, we also analyze a linear demand case to examine the magnitude of the problem. With linear demand, we find that whenever the incumbent can induce one buyer to sign a loyalty discount contract, all buyers sign and loyalty discounts are completely exclusionary. With only three buyers, loyalty discounts with buyer commitment can profitably exclude the rival from the market whenever the entrant's cost advantage is less than 10% and the incumbent's costs are less than 28% of the choke price. With twenty buyers, the same result can be achieved as long as the incumbent's costs are less than 80% of the choke price. In these cases, all buyers pay the incumbent's monopoly price.

We show that loyalty discounts *without* buyer commitment, we prove that they can create anticompetitive effects through market division rather than through exclusion. Because the incumbent's loyalty discount requires it to charge loyal buyers less than buyers who are not covered by the loyalty discount (uncovered buyers), the incumbent cannot lower prices to uncovered buyers without also lowering prices to loyal buyers. This makes it more costly for it to compete for uncovered buyers and effectively cedes those buyers to the entrant, which reduces the entrant's incentive to compete aggressively for covered buyers.

We prove that, if the entrant's cost advantage is not too large, loyalty discounts without buyer commitment soften competition and increase prices above competitive levels, reducing consumer and total welfare, although they cannot exclude a more efficient entrant from more than half the market. Thus, loyalty discounts without buyer commitment can also be anticompetitive even if they cover a minority of buyers. Further, these anticompetitive effects result even if the entrant is more efficient, and all loyalty discounts are above cost.

Our paper is related to the literature on ordinary exclusive dealing. Rasmusen et al. (1991) and Segal and Whinston (2000) showed that exclusive dealing can deter entry if there are many buyers and economies of scale in production.[6] Simpson & Wickelgren (2007) extend these models to find that, even without economies of scale, sellers can still get buyers to accept an anticompetitive exclusionary agreement in exchange for a small side payment if the buyers are intermediaries.[7]

Ordover and Shaffer (2007) and Elhauge (2011) have concluded that loyalty discounts

[6]Other articles that find exclusive dealing can have anti-competitive effects include Aghion and Bolton (1987), Mathewson and Winter (1987), Spier and Whinston (1995), Bernheim and Whinston (1998), and Neeman (1999). Innes and Sexton (1994) argue that the Chicago School claim that exclusive contracts are necessarily efficient can be resurrected if one allows all the players to form coalitions and price discrimination is prohibited. Papers that have related loyalty discounts to exclusive dealing include See Tom, et al. (2000); Elhauge (2003); Spector (2005); Whinston (2006); Kaplow & Shapiro (2007); and Elhauge (2008)

[7]Argenton (2010) shows that a lower quality incumbent can deter the entry of a higher quality rival in this situation as well. For a contrary view, see Fumagalli and Motta (2006). Chen and Shaffer (2010) show that partially-exclusive contracts can sometimes lead to exclusion with positive probability when (in a coalition-proof equilibrium) there would be no exclusion with fully exclusive contracts. They do not analyze loyalty discounts that involve the crucial feature of a seller commitment to charge loyal customers less than disloyal ones, nor do they consider loyalty discounts without buyer commitment. Also related is the literature that shows that exclusivity agreements can restrain competition even if they do not preclude entry. Rey and Stiglitz (1995) show this for exclusive territories. Lin (1990) shows this for exclusive dealing contracts that require each manufacturer to use its own retailer. Wright (2008) shows that if an upstream entrant is sufficiently differentiated from the incumbent, the incumbent will sign exclusive deals with all but one retailer to allow the entrant to enter but force its price to be high through double marginalization.

without buyer commitment can create anticompetitive effects when there exist switching costs, financial constraints, limits on rival expandability, or buyer demand that is segmented into contestable and incontestable portions that can be effectively bundled by the loyalty discount.[8] Our analysis shows that those assumptions are unnecessary to show anticompetitive effects given the seller commitment used in loyalty discounts.

Our analysis has some similarities with prior literature showing that switching costs can diminish incumbent incentives to lower its market price to compete for new buyers (who have no cost of switching to the entrant) because doing so would reduce profits on sales to existing customers, which in turn can give entrants incentives to price in a way that competes only for new buyers.[9] The difference is that sellers can adjust the size of the loyalty discount and number of buyers who get it to maximize the anticompetitive effects.

Some argue that if loyalty discounts involve seller commitments on pricing, then the analysis is the same as for price-matching clauses or most favored nation clauses.[10] However, both the commitments and effects differ sharply. While price-matching and most favored nation clauses involve seller pricing commitments, they do not involve seller commitments to charge loyal buyers less than other buyers. Price-matching clauses involve seller commitments to match rival prices and are not conditioned on loyalty. Loyalty discounts have the *opposite* effect of *discouraging* sellers from matching rival prices for uncovered buyers. Most-favored-nations clauses involve seller commitments to charge agreeing buyers no more than the seller charges other buyers and are not conditioned on loyalty. In contrast, loyalty discounts involve seller commitments to charge loyal buyers affirmatively *less* than the seller charges other buyers. We find that the profit-maximizing loyalty discount typically exceeds zero. Further, we find that loyalty discounts have anticompetitive effects under very different conditions than are assumed in the literature on most-favored nation clauses.[11]

[8]Marx and Shaffer (2004) and Kolay et al. (2004) also examine market-share and all-units discounts, but without considering the effects of a seller commitment to maintain a loyalty discount.

[9]See Deneckere et al. (1992), Padilla (1992) Narasimhan (1988), Klemperer (1987), Farrell and Shapiro (1988), Gabszewicz et al. (1992), and Wang and Wen (1998). For an excellent survey of the switching cost literature, see Farrell and Klemperer (2007).

[10]See, e.g., Lambert (2012); Wright (2008).

[11]Most articles on most favored nations clauses find anticompetitive effects under oligopolistic coordination, see Cooper (1986), or if a monopolist is selling a durable good (Butz 1990; Marx & Shaffer 2004). Hviid and Shaffer (2010) find that a most favored nation clause can have anti-competitive effects in a one-shot game if it is combined with a price-matching clause. None of those assumptions is necessary to show anticompetitive

Our paper is most similar to Elhauge (2009), which did analyze loyalty discounts with and without buyer commitments and likewise found that they can have anticompetitive effects even without economies of scale. However, this paper differs from his article in several important ways that lead to different and more general results. First, his article assumes linear demand and does not allow the potential entrant to be more efficient than the incumbent seller. We prove our basic results without imposing any specific functional form on buyer demand (other than weak concavity), and we allow the rival's marginal cost to be less than the incumbents. This formulation also allows us to explore, in our linear demand simulations, how much more efficient an entrant can be and still be foreclosed by loyalty discount contracts. It also allows us to investigate the profit-maximizing size of the loyalty discount, which he did not analyze.[12]

Second, Elhauge (2009) assumes sequential rather than simultaneous pricing when determining whether buyers will accept loyalty discount contracts.[13] As a result, he finds there always exists an equilibrium in which loyalty discounts with buyer commitments have no effect because all buyers reject them.[14] In contrast, by considering simultaneous pricing, we find that given a sufficient number of buyers (in our linear demand analysis, "sufficient" can mean three buyers), there does not exist an equilibrium in which all buyers reject a loyalty discount with buyer commitments.

Our analysis of loyalty discounts without buyer commitment differs from Elhauge (2009) in two ways that changes the results. (A) Elhauge never formally analyzed the profit-maximizing fraction of covered buyers without economies of scale. We find that the profit-maximizing fraction is always less than half when the entrant is more efficient. This distinction has important policy relevance because it means that loyalty discounts without buyer commitment can, without economies of scale, have anticompetitive effects even if they cover a minority of the market. (B) Because he does not analyze the profit-maximizing fraction

effects from loyalty discounts under the model offered here.

[12]His lack of analysis of how the loyalty discount would be set has been a subject of critique by some. *See* Eilat, et al. (2010).

[13]He considers simultaneous pricing (along with sequential pricing) in determining the equilibrium in the pricing subgame with a given fraction of buyers signing a loyalty discount contract, but assumes sequential pricing when analyzing whether buyers will accept the loyalty discount.

[14]This was another basis for critique of the Elhauge model in Eilat, et al. (2010).

of covered buyers, he does not correctly establish the prices that will be paid. We show that while loyalty discounts without buyer commitment do enable the incumbent to take market share from a more efficient entrant and elevate prices above the competitive level, they do not lead to monopoly pricing with certainty, contrary to Elhauge (2009).

Section 2 outlines our model of loyalty discounts with buyer commitment. In section 3, we present our general analysis of the model. In 3.1 we examine the duopoly pricing equilibrium. In 3.2 we consider the buyers' decisions to sign loyalty discounts. Section 4 contains numerical analysis of the linear demand case, with 4.1 examining the conditions for exclusion and 4.2 analyzing the profit-maximizing loyalty discount. Section 5 analyzes loyalty discount without commitment. Section 6 concludes. Proofs not in the text, along with some numerical analysis, are in the appendix.

# 2 Model

An incumbent firm, $I$, produces a good at constant marginal cost, $c$. The entrant, $E$, produces the same good at marginal cost, $c_e < c$. We assume, to highlight the anticompetitive effects of loyalty discounts, that $E$ incurs no fixed cost to enter the market.[15] There are $N$ buyers. Each buyer's demand function is $q(p), q' < 0, q'' \leq 0$. Let $p^m = \arg\max_p (p-c)q(p)$; $p^m$ is $I$'s profit-maximizing monopoly price. Let $p_e^m = \arg\max_p (p - c_e)q(p)$; $p_e^m$ is $E$'s profit-maximizing monopoly price. To simplify the exposition, let $\pi_I(p) = (p - c)q(p)$; $\pi_e(p) = (p - c_e)q(p)$; $\pi_I^m = (p^m - c)q(p^m)$; $\pi_e^m = (p_e^m - c_e)q(p_e^m)$

Let $s(p)$ be a buyer's consumer surplus from buying $q(p)$ of the good at price $p$. $s(p) = \int_p^\infty q(x)dx$. We make the following assumption about the maximum size of $E$'s cost advantage throughout the model:

**Assumption (*)** $\pi_I^m > s(p_e^m) - s(p^m)$

Assumption (*) means that $I$'s monopoly profit exceeds the increase in consumer surplus that results when buyers purchase at $E$'s monopoly price rather than $I$'s. Since $s(p_e^m) - s(p^m)$

[15] What is essential is that the incumbent can offer buyers loyalty discount contracts before the entrant can compete on price.

is strictly decreasing in $c_e$ and is zero at $c_e = c$, this assumption requires that $c_e$ is not too far below $c$. This assumption means that our analysis applies as long as $E$ is not too much more efficient than $I$.

We have three periods in our model. In period 1, $I$ offers a non-discriminatory loyalty discount contract to buyers of the form $\{d, t\}$ in which buyers commit to buy only from $I$ in exchange for receiving a discount of $d$ off the price that $I$ offers to buyers who do not sign the contract and a transfer of $t$ from $I$ to the buyer. $E$ is not present at this time. Buyers individually, but simultaneously, decide whether or not to accept this offer in period 1. Those who accept must buy from $I$ in period 3 and receive the discount. Thus, for any price $p_f$ that $I$ offers to free buyers who made no loyalty commitment, any buyer who did accept a loyalty commitment in period 1 receives a price of $p_f - d$. Let $\theta$ be the share of buyers who agree to the loyalty discount.

In period 2, $E$ and $I$ set prices simultaneously to free buyers of $p_e$ and $p_f$ respectively. $I$'s price to captive buyers is $p_f - d$. If $\theta = 0$, because $E$ and $I$ produce identical products, we have the standard Bertrand result that $E$ captures the entire market at a price of $p_e = c$.

In period 3, buyers make purchase decisions. If $p_e \leq p_f$, free buyers purchase only from $E$, and committed buyers purchase entirely from $I$.[16] Thus, $I$'s profit is $\theta\pi_I(p_f - d)$, while $E$'s profit is $(1-\theta)\pi_e(p_e)$. If $p_e > p_f$, then all buyers purchase from $I$. Thus, $I$'s profit is $\theta\pi_I(p_f - d) + (1-\theta)\pi_I(p_f)$ while $E$'s profit is zero.

Notice that in period 1 $I$ only offers a discount off the price it will later offer to buyers who did not agree to the loyalty discount ($I$ does not commit to a price when it offers the loyalty discount). This is a fairly common assumption in the exclusive dealing literature. This could be because the good is hard to describe in period 1. It could be because the cost of production is not precisely known in period 1 (our model could easily accomodate a common shock to production costs). Furthermore, in this model, committing just to a price to loyal buyers rather than a loyalty discount is not as profitable for $I$.

What is critical is that $I$ can structure its loyalty discount contract in a way that limits

[16] Notice, we make the standard tie-breaking assumption that if both firms charge the identical price, then buyers buy from the lower cost firm ($E$ in this case). This assumption makes the analysis of the standard Bertrand equilibrium less cumbersome since the equilibrium price is simply the cost of the higher cost firm rather than some epsilon below that level.

its incentive to compete for disloyal buyers, thereby reducing the payoff to rejecting the contract. The loyalty discount we identify has this property. A percentage discount, rather than a fixed dollar discount, would also have this property. Giving $I$ the option to specify a price that also includes a loyalty discount commitment would create a similar effect. That is, consider $I$'s period 1 offer of the form $\{p_c, d, t\}$, where this means that the price a committed buyer pays in period 3 is $Min\{p_c, p_f - d\}$. If $I$ sets $p_c = p^m$, the equilibrium will be identical to the one we study because $p_f - d \leq p^m$ in our equilibrium with probability one. As long as $I$ can set price in period 1, then there can be no equilibrium that gives it lower profits than the one we study, which rules out the competitive equilibrium in which $I$ earns zero profit. This does require that $I$ can make a take it or leave it price offer.

# 3 General Analysis

## 3.1 Duopoly Pricing Equilibrium

In the period 2 pricing subgame, if $\theta$ is large enough (and $d > 0$ or $c_e < c$), then there is a unique pure-strategy equilibrium in which both firms charge their monopoly prices ($p_f = p^m + d$ and $p_e = p_e^m$), committed buyers purchase from $I$ at $p^m$ and free buyers purchase from $E$ at $p_e^m$. $E$ has no incentive to deviate because it is maximizing its profits. $I$ could undercut $E$ by charging $p_f = p_e^m - \varepsilon$ for some $\varepsilon > 0$. For large enough $\theta$, the reduction in profits from committed buyers outweighs the gain from stealing free buyers. To show that the equilibrium is unique, imagine there is another equilibrium with $p_f \neq p^m + d$. If $p_f > p^m + d$, then $I$ could easily increase its profits by reducing $p_f$ to $p^m + d$. If $p_f < p^m + d$, even if $I$ captured the entire market, the above argument shows that for $\theta$ large enough, $I$ can increase its profits by setting $p_f = p^m + d$ and only selling to committed buyers.

To determine the minimum $\theta$ for this equilibrium, notice that $I$ is indifferent between selling to captive consumers at $p^m$ undercutting $p_e^m$ to sell to all consumers if and only if

$$\bar{\theta}\pi_I(p_e^m - d) + (1 - \bar{\theta})\pi_I(p_e^m) = \bar{\theta}\pi_I^m \qquad (1)$$

If $q(p_e^m - d) = \infty$, for large but finite $d$, then $\bar{\theta}$ will not exist. We guarantee existence by assuming that $q(p) < \infty$ for any finite $p$ (even $p < 0$), which is reasonable with large and convex disposal costs. For $\theta \geq \bar{\theta}$, there is a unique pure-strategy equilibrium in which $I$ and $E$ charge monopoly prices ($p^m$ and $p_e^m$) to committed and free buyers respectively.

For $\theta < \bar{\theta}$, there is no pure strategy equilibrium. To see this, define $p_e^*$ implicitly by:

$$\theta\pi_I(p_e^* - d) + (1 - \theta)\pi_I(p_e^*) = \theta\pi_I^m \tag{2}$$

Obviously, $p_e^*$ is a function of $\theta$. For $p_e > p_e^*$, $I$'s best response is to choose $p_f = p_e - \varepsilon$, while for $p_e \leq p_e^*$, $I$'s best response is to choose $p_f = p^m + d$. But if $p_f = p_e - \varepsilon$, then $E$'s best response is not to charge $p_e$ (because that yields zero profit), but to charge $p_f$ (recall that all sales go to $E$ with equal prices). If $p_f = p^m + d$, then $E$'s best response is to charge $p_e^m > p_e^*$ for $\theta < \bar{\theta}$. We have now proved the following lemma.

**Lemma 1** *For $\theta \geq \bar{\theta}$, the unique pure-strategy equilibrium in the period 2 pricing subgame is for $I$ to set $p_f = p^m + d$ and $E$ to set $p_e = p_e^m$. Then $I$ sells to committed buyers at $p^m$ and $E$ sells to free buyers at $p_e^m$. For $\theta < \bar{\theta}$, there is no pure-strategy equilibrium in the period 2 subgame.*

#### 3.1.1 Deriving the Mixed Strategy Equilibrium

We now characterize the mixed strategy equilibrium for $\theta < \bar{\theta}$.[17] Let $F(p)$ ($F_e(p)$) denote the cumulative distribution function of $I$'s ($E$'s) price to free buyers . $I$'s flow profit (ignoring transfers from loyalty contracts) is:

$$\theta\pi_I(p_f - d) + (1 - \theta)(1 - F_e(p_f))\pi_I(p_f) \tag{3}$$

$I$ sells to all committed buyers at a price of $p_f - d$. It also sells to the $1 - \theta$ free buyers if $E$'s price is greater than $p_f$. This occurs with probability $1 - F_e(p_f)$.

[17] Of course, because $\bar{\theta}$ depends on $d$, $I$ can influence how many buyers it takes to create this pure strategy equilibrium. The larger is $d$, the smaller is the $\bar{\theta}$ needed to generate the monopoly pricing equilibrium. As we will see below, a very large $d$ also makes it more costly to induce buyers to sign a loyalty discount contract if they believe other buyers will not do so.

For $I$ to play a mixed strategy, (3) must be the same for all $p_f$ in the support of $F$. The following lemma provides a key property of $I$'s pricing strategy.

**Lemma 2** *In any mixed strategy equilibrium, $I$ chooses $p_f = p^m + d$ with positive probability.*

Proof. Because $E$ will never charge above $p_e^m$, $I$ will never choose $p_f \in [p_e^m,\ p^m + d)$ because $I$ would be charging committed buyers less than $p^m$ while making no sales to free buyers. If $I$ never charges $p^m + d$, then $I$'s maximum price must be strictly below the maximum price in the support of $E$'s strategy. In that case, $E$'s strategy cannot be a best response to $I$'s strategy, so this cannot be an equilibrium. Q.E.D.

Lemma 2 implies that $I$'s profits from any price (expression (3)) must equal $\theta\pi_I^m$. Solving for $F_e$ over the range in which the distribution is atomless gives:

$$F_e(p) = 1 - \theta\frac{\pi_I^m - \pi_I(p-d)}{(1-\theta)\pi_I(p)} \tag{4}$$

Notice, however, that using (4), $E$'s distribution function would not reach one until a price of $p^m + d$. But, $E$ has no reason to ever charge above $p_e^m$. Thus, there is an atom in $E$'s pricing distribution at $p_e^m$ of $\theta\frac{\pi_I^m - \pi_I(p_e^m - d)}{(1-\theta)\pi_I(p_e^m)}$. We denote the bottom of the support of $E$'s pricing distribution by $p_0$, which is defined implicitly by

$$\theta\pi_I(p_0 - d) + (1-\theta)\pi_I(p_0) = \theta\pi_I^m \tag{5}$$

$E$'s complete pricing distribution function is:

$$F_e(p) = \begin{cases} 1 - \theta\frac{\pi_I^m - \pi_I(p-d)}{(1-\theta)\pi_I(p)} & \text{for } p_0 \leq p < p_e^m \\ 1 & \text{for } p \geq p_e^m \end{cases} \tag{6}$$

To determine $I$'s pricing distribution function, we look at $E$'s profit function:

$$(1-\theta)(1 - F(p_e))\pi_e(p_e) \tag{7}$$

$E$ only sells to the $1-\theta$ free buyers and only if $I$'s price exceeds $E$'s price. This occurs with probability $1 - F(p_e)$. $E$'s mixed strategy requries that it receives the same profits for all prices in the support of $F_e$. The minimum price in $I$'s distribution is also $p_0$, the price for which $I$ earns as much profit from capturing the entire market (charging free buyers $p_0$ and committed buyers $p_0 - d$) as from selling only to committed buyers at $p^m$. The minimum of its pricing support cannot exceed this level because if it did, then $E$ would price above $p_0$ with probability one. In that case, $I$ could increase its profits by charging free buyers just under $E$'s minimum price so as to capture the entire market while charging free buyers greater than $p_0$. Using (5) and (7), we can solve for $F$:

$$F(p) = 1 - \frac{\pi_e(p_0)}{\pi_e(p)} \tag{8}$$

Using (8), $I$ has a positive probability of choosing a price to free buyers that exceeds $p_e^m$. We know that the only price above $p_e^m$ that $I$ ever charges is $p^m + d$. Thus, there is an atom at $p^m + d$ of $\frac{\pi_e(p_0)}{\pi_e^m}$, so that the complete distribution function for $I$'s price to free buyers has support of $[p_0, p_e^m] \cup \{p^m + d\}$ and is given by:

$$F(p) = \begin{cases} 1 - \frac{\pi_e(p_0)}{\pi_e(p)} & \text{for } p_0 \leq p < p_e^m \\ 1 - \frac{\pi_e(p_0)}{\pi_e^m} & \text{for } p \in [p_e^m, p^m + d) \\ 1 & \text{for } p \geq p^m + d \end{cases} \tag{9}$$

### 3.1.2 Properties of the Mixed Strategy Equilibrium

Given $\theta$ and $d$, the pricing equilibrium is given by (6) and (9) for $\theta < \bar{\theta}$. Part A of Proposition 1 shows how the pricing equilibrium changes as the fraction of signed buyers increases from zero to $\bar{\theta}$. Part B derives some additional comparative statics.

**Proposition 1**

**A** *(i) As* $\theta \to 0$, $\Pr[p_e = c] \to 1$; *that is, as the fraction of signed buyers approaches zero the probability that E prices at I's costs approaches one. (ii) As* $\theta \to \bar{\theta}$, $\Pr[p_e = p_e^m] \to 1$ *and* $\Pr[p_f = p^m + d] \to 1$; *that is, as the fraction of signed buyers approaches the pure strategy equilibrium level, the probability that both E and I choose their monopoly prices approaches one. (iii) If* $p < p_e^m$, $\frac{dF_e(p)}{d\theta}, \frac{dF(p)}{d\theta} < 0$; *the probability that both E and I choose a price lower than any given level is strictly decreasing in fraction of signed* *buyers, which implies that the average price that both firms choose is strictly increasing the fraction of signed buyers.*

**B** *(i) Increasing $c_e$ to $c'_e$ has no affect on E's pricing distribution below $p_e^m$ but creates a positive probability of prices $p_e \in (p_e^m, p_e'^m]$. Increasing $c_e$ to $c'_e$ increases the probability that I prices lower than any given level $p < p_e^m$ but creates a positive probability of prices $p \in (p_e^m, p_e'^m]$. (ii) Increasing c reduces the probability that both E and I price lower than any given level $p < p_e^m$. (iii) Increasing d reduces the probability that both E and I price lower than any given level $p < p_e^m$.*

Proof. See Appendix.

Part A tells us that if the fraction of signed buyers is very low, the equilibrium will look a lot like the standard Bertrand equilibrium where $E$ prices at the incubment's costs and captures the entire market. But, as the fraction of signed buyers increases, prices go up (in a first order stochastic dominance sense) and eventually approach the pure strategy equilibrium when the number of buyers is large enough in which both firms charge their monopoly prices and serve their respective parts of the market.

Part B provides some comparative statics for other parameters in the model. Because $E$'s price distribution is set to keep $I$'s profit constant, changes in $E$'s costs only affect $I$'s pricing strategy. Because $I$ has the option of selling to captive buyers at its monopoly price, when $I$'s cost increase, it becomes less profitable for it to compete for free buyers. This makes $E$ price less aggressively to maintain $I$'s mixed strategy. $I$'s minimum profitable price increases with its costs, making its price distribution less aggressive. A larger discount has the same effect because it makes it more costly for $I$ to compete for free buyers by reducing its profit from captive buyers.

## 3.2 Buyer Loyalty Discount Decisions

Given the pricing distribution functions, we determine a buyer's decision to accept $I$'s loyalty discount offer. Proposition 2 gives our main result for loyalty discounts with commitment.

**Proposition 2** *A. For any $d \geq 0$, and for any $c$ and $c_e$, the minimum $t$ necessary to induce at least one buyer to sign a loyalty discount contract is strictly decreasing in N. If N* *is sufficiently large and $c - c_e$ is small enough, then there exists a $d \geq 0$ and $t \geq 0$ for which $I$ can profitably offer buyers a loyalty discount contract of $\{d, t\}$ such that, in any equilibrium in the subgame following this offer, some buyers accept this offer and prices exceed what they would have been absent loyalty discounts.*

B. *For any $N \geq 2$, there exists a $d \geq 0$ and $t \geq 0$ for which $I$ can profitably offer buyers a loyalty discount contract of $\{d, t\}$ such that there exists an equilibrium in the subgame following this offer such that all buyers accept $I$'s offer, $E$ is excluded from the market, and all buyers purchase from $I$ at $p^m$. As $N$ increases, the minimum $d$ necessary for this equilibrium decreases.*

Proof. First, we prove part B. If buyers expect that, even after rejecting this contract, $\theta \geq \bar{\theta}$, then they will pay $p^m$ if they accept and $p_e^m$ if they reject. If $N - 1$ buyers will accept, then $\theta \geq \bar{\theta}$ even if one buyer rejects if

$$\frac{N-1}{N} \geq \frac{\pi_I(p_e^m)}{\pi_I(p_e^m) + \pi_I^m - \pi_I(p_e^m - d)} \tag{10}$$

Since $\pi_I^m > \pi_I(p_e^m)$, the right hand side is less than one-half if $d \geq p_e^m - c$, which means that with only two buyers, it only takes one signing buyer to induce each firm to charge its monopoly price. Because $\pi_I^m > s(p_e^m) - s(p^m)$, there exists a $t \in ([s(p_e^m) - s(p^m), \pi_I^m]$ such that all buyers will accept $\{d, t\}$ for such a $t$ if they believe enough other buyers will (such that $\theta \geq \bar{\theta}$ even if they reject). Because all buyers accept the contract, $I$ can profitably offer this contract with $t \leq \pi_I^m$. $E$ is excluded and $I$ charges all buyers $p^m$. Since the right hand side of (10) is strictly decreasing in $d$ for $d \geq 0$, as $N$ increases, increasing the left hand side, the inequality can be satisfied for a smaller $d$.

To prove part A, we show that a buyer signs even if she believes all other buyers will reject. Because $I$'s distribution function is decreasing in $\theta$, expected consumer surplus as the only signer is increasing in $N$ (because $\theta = 1/N$ if a buyer is the only signer). The consumer surplus from rejecting is independent of $N$ (price is fixed at $c$). This makes the amount necessary to compensate one buyer for signing the loyalty discount decreasing in $N$.

If all buyers reject, each buyer obtains a surplus of $s(c)$. If $N - 1$ buyers reject, then the one signer can purchase only from $I$ at $p_f - d$. Its surplus from accepting depends on

$F(p_f)$, which depends on $N$ because $p_0$ depends on $\theta = 1/N$. As $N$ grows arbitrarily large, $\theta \to 0$. As $\theta \to 0$, $p_0 \to c$.

The probability that $I$ charges above $p$ is $\frac{\pi_e(p_0)}{\pi_e(p)}$ (for $p < p_e^m$). Because $p_0$ can be arbitrarily close to $c$, this probability can be arbitrarily close to zero for $p > c$ if $c_e = c$. So, if $E$ has no cost advantage, as $N$ gets large, $I$ will charge a price arbitrarily close to $c$ with probability arbitrarily close to one. So, even if $d = 0$, the committed buyer will obtain a consumer surplus arbitrarily close to $s(c)$. Thus, if $c_e = c$, if a buyer thinks all other buyers will not sign, this buyer can be induced to sign a loyalty discount contract for an arbitrarily small amount. The construction of the mixed strategy equilibrium guarantees that $I$'s expected profit per signer is $\pi_I^m >> 0$ because $I$ could charge this buyer the monopoly price. Furthermore, because $F$ is continuous in $c_e$, for $c - c_e$ small enough, the buyer's expected surplus loss from signing the loyalty discount contract will remain less than $\pi_I^m$. Thus, if $c - c_e$ is small enough, $I$ will always offer a payment large enough to induce at least one buyer to sign rather than earn zero when all buyers are free. With a positive number of committed buyers, prices will exceed $c$ with positive probability. Q.E.D.

**Corollary** *If $c - c_e$ is small enough, then there is an equilibrium in the full game in which $I$ offer buyers a loyalty discount contract of $\{d, t\}$ that all buyers accept, $E$ is excluded, and all buyers purchase from $I$ at $p^m$.*

Proof. As $c - c_e \to 0$, then $s(p_e^m) - s(p^m) \to 0$, which means the $t$ necessary to induce buyers to agree given that all other buyers will agree is arbitrarily small. This ensures there is no other more profitable offer that $I$ can make in which not all buyers agree to the loyalty discount contract. Q.E.D.

This corollary says that if $E$'s cost advantage is small, there is an equilibrium in which $I$'s optimal strategy is to offer a loyalty discount contract that locks up the entire market. We cannot say that it is the unique equilibrium because this equilibrium relies on buyers believing that other buyers will accept the contract.

Proposition 2A, however, says that even though there might be other equilibria based on different buyer beliefs, none of them can involve all buyers rejecting the loyalty discount contract. This occurs even though these loyalty discounts raise prices above the competitive level and make all buyers worse off. While prior results (Rasmusen et al. 1991, Segal and Whinston 2000) on exclusive dealing contracts established that these contracts can create *an* equilibrium in which all buyers sign exclusive dealing contracts and buyers pay supra-competitive prices as a result, Proposition 2 demonstrates that loyalty discount contracts are more damaging for two reasons. First, for basic exclusive dealing contracts without discrimination, there is always an equilibrium in which every buyer rejects the contract and all buyers pay competitive prices. In fact, this is the unique perfectly coalition proof Nash equilibrium. In contrast, Proposition 2 shows that with loyalty discount contracts, such an equilibrium does not always exist. With enough buyers, if $I$'s costs are not too far above $E$s, then some buyers must always sign the loyalty discount contracts because the expected increase in price to the signing buyer is very small if very few buyers sign, so the cost to compensate this buyer is less than the added profit from having one more committed buyer. This results in all buyers paying supra-competitive prices.

Second, the prior results on exclusive dealing, at least when buyers are final consumers or independent monopolists, only demonstrated anticompetitive results in the presence of economies of scale. Proposition 2 shows that loyalty discounts will have anticompetitive results even with constant returns to scale.

The reason that loyalty discounts have such robust anticompetitive effects is that they create a negative externality among buyers even without economies of scale. Every buyer who signs a loyalty discount contract changes both $I$'s and $E$'s pricing distributions by making them less aggressive. Thus, a buyer who signs a loyalty discount contract passes some of the social loss from doing so onto other buyers.

While there is always an equilibrium with complete exclusion and never an equilibrium with no exclusion, there could be an equilibrium in which an intermediate number of buyers sign a loyalty discount contract. This would not result in complete exclusion, but it does lead to prices that are above the competitive level with positive probability. While Proposition 2 does not rule out this possibility, in our numerical analysis we did not find any cases in which loyalty discounts with commitment result in only partial exclusion.

Proposition 2 says that anticompetitive effects are guaranteed if there are enough buyers and $E$'s cost advantage is small enough. Even with a general demand function, we can go further to say that the effect of $E$'s cost and the number of buyers on the ability of $I$ to get one signer is monotonic.

**Remark 1** *(i) If, for any given $c$ and $N$, $I$ can profitably induce at least one buyer to sign a loyalty discount contract in period 1 if $E$'s costs are $c_e$, then $I$ can induce at least one buyer to sign a loyalty discount contract in period 1 if $E$'s costs are $c_e > c_e$. (ii) If, for any given $c$ and $c_e$, $I$ can profitably induce at least one buyer to sign a loyalty discount contract in period 1 if the there are $\bar{N}$ buyers, then $I$ can induce at least one buyer to sign a loyalty discount contract in period 1 if there are $N > \bar{N}$ buyers.*

Proof. See Appendix.

Before proceeding, let's pause to consider the robustness of our results to the buyer's option to breach the loyalty discount contract and pay damages. While we will not do a complete analysis here, it is worth noting that the result of the mixed strategy equilibrium will sometimes be that $I$'s loyalty discounted price will be lower than $E$'s price. In those situations, there is no incentive for a buyer to breach. Even in situations in which $E$'s price is lower than the loyalty discounted price, $E$'s price will generally exceed its costs. This means that even if total surplus is higher with breach, the joint surplus of $I$ and buyer may not be higher with breach because breach transfers profit to $E$. Lastly, note that even if a buyer does breach with positive probability, if the buyer must pay expectation damages, $I$ can still earn the same level of profit.[18] Thus, the possibility of breach will not reduce the amount $I$ will pay buyers to sign the loyalty discount contracts. Furthermore, it does not change the fact that a buyer externalizes some of its costs of signing the contract because signing the contract will induce $I$ to price less aggressively even if it expects breach.

# 4 Numerical Analysis–Linear Demand Case

With general demand functions, we have shown that with enough buyers and a small enough entrant cost advantage, loyalty discounts soften competition in any equilibrium. To examine

[18] Further, in some circumstances, such as when rebates are denied, a breaching buyer will have to pay more than expectation damages (Elhauge 2009 p.205-6).

how many buyers is enough or how small the cost advantage must be, we assume a linear demand function, $q(p) = 1 - p$. This generalizes to any linear demand function because any function $a - b\tilde{p}$ is equivalent to a demand function of $1 - p$ for $a$ units of the good priced in monetary unit that is $a/b$ of the original unit. Because the details of this analysis is long and not particularly enlightening, we leave it to the appendix. Note that even with linear demand, analytic solutions are not feasible, so we use numerical analysis.

## 4.1 Numerical Analysis–Conditions for Exclusion

Through numerical analysis, we are able to find the minimum value for $E$'s marginal cost, $c_e$, for which $I$ can profitably induce all buyers to sign a loyalty discount contract (given the least $I$-friendly buyer beliefs about what other buyers will do) for any given level of $I$'s cost, $c$, and the number of buyers, $N$ (the details are in the web appendix). We do this for every integer value of $N$ between 3 and 20 (for $N = 2$ we observed that it is not possible to induce one buyer to sign a loyalty discount if she believes the other buyer will not sign even if $c_e = c$) and for $c \in \{.1, .2, ..., .9\}$. Then, we use linear regression to find an expression that best predicts this minimum value of $c_e$.

$$\begin{aligned} \hat{c}_e &= 0.2080 + 0.7594c + 0.0361c^2 + 0.0066N - 0.0102cN + 0.0040c^2N \\ &\quad -0.2408Log[N] + 0.2736cLog[N] - 0.0362c^2Log[N] \end{aligned} \tag{11}$$

All of these coefficients have p-values of less than .01. The R-Squared is .99999.

Figure 1 uses (11) to plot $\hat{c}_e/c$ as a function of $c$ with $N = 3, 7$, and 20. With only three buyers,loyalty discounts completely foreclose $E$ if the ratio of $E$'s cost to $I$'s costs is above the thin solid line. When the number of buyers rises to seven, loyalty discounts can profitably exclude $E$ for a much larger range of costs (above the dashed line). As the figure shows, with 20 buyers (thick line), foreclosure under any beliefs is even easier, especially if $I$'s costs are not too large.

Figure 2 depicts the effect of increasing the number of buyers by using (11) to plot $\hat{c}_e/c$ as a function of $N$ with $c = 0.2, 0.5$, and 0.8. Increasing the number of buyers allows $I$ to completely foreclose $E$ under any set of buyer beliefs even as $E$'s cost advantage increases. The magnitude of the effect of increasing the number of buyers is diminishing as the number of buyer's increases and is more pronounced if $I$'s costs are lower.

With 20 or fewer buyers, the maximum value for the minimum $c_e$ always occurred when buyers believed that no other buyer would sign the loyalty discount contract. That is, if $I$ can profitably get one buyer to sign a loyalty discount contract then it can profitably get all buyers to do so. It is not the case, however, that inducing the first signer is always the most costly. For example, if $c_e$ is substantially above the minimum value necessary for there to exist an equilibrium in which no buyers sign the loyalty discount contract, then it can be most costly for $I$ to induce signing where buyers believe that some, but not all, other buyers will sign the loyalty contracts. *Nonetheless, in every case that we examined, it was always most profitable to sign all the buyers rather than to pay a smaller amount and sign less than all buyers. That is, we never found a case in which there was an equilibrium in which some, but not all, buyers sign the loyalty discount contract.*

Our linear demand results make four main points that demonstrate that the limiting results in Proposition 2 likely have significant practical impact. These are summarized in the following remark.

**Remark 2** *With linear demand and 20 or fewer buyers:*

**1.** *If I can profitably induce one buyer to sign a loyalty discount contract then it can profitably create a unique equilibrium in which all buyers sign a loyalty discount contract.*

**2.** *If E's cost advantage is small, the unique exclusionary equilibrium occurs with three buyers.*

**3.** *The larger the potential market (based on demand when price equals cost), the easier it is for I to create this unique exclusionary equilibrium.*

**4.** *I can create a unique exclusionary equilibrium for a substantial range of parameters even if E has a non-trivial cost advantage. For example, if E has no more than a 10% cost advantage, I can create a unique exclusionary equilibrium with three buyers if its costs* *are less than 28% of the choke price, with seven buyers if its costs are less than 68% of the choke price, and with 20 buyers if its costs are less than 80% of the choke price.*

In our model, with zero costs of entry, competition is always efficient. Thus, our linear demand results show that there is substantial scope for $I$ to profitably use loyalty discounts to foreclose efficient competition, reducing both productive efficiency and consumer surplus. Notice that this occurs, unlike in prior models of exclusive dealing, without any economies of scale and even with buyers with independent demands.

## 4.2 Numerical Analysis–Optimal Loyalty Discount

Characterizing the optimal also requries numerical analysis. Our numerical analysis produced the following simple regression equation that allows us to estimate the optimal loyalty discount given the costs of $E$ and $I$ and the number of buyers (R-squared of .901):

$$\hat{d} = 0.023 - 0.091c - 0.430c_e + 0.201Log[N] \tag{12}$$

All coefficients are significant at the .05 level and the coefficients of $c_e$ and $Log[N]$ are significant at well-beyond the .001 level. This tells us that $I$ offers contracts with higher discounts the more firms there are.[19] Increasing both $E$'s and $I$'s costs result in lower discounts, though the effect is much stronger for changes in $E$'s costs. It is worth remembering that a larger discount is not necessarily good for the signing buyer because it leads $I$ to price less aggressively. To get a sense of the magnitude of the discount, note that if $I$'s costs are 0.5 and $E$'s are 0.45 (a 10 percent cost advantage), if there are 10 buyers, then the optimal discount is about 0.25, which is one-third of $I$'s profit-maximizing monopoly price, or about one-quarter of the optimal monopoly price before the discount.[20] Figure 3 gives a graph of the optimal discount for three different levels of $I$'s costs when $E$ has a ten percent cost advantage as a function of the number of buyers. For $c = 0.7$, the optimal discount is

[19]Recall, that this result does depend on buyers making the worst-case assumption that the number of other buyers who will agree to the loyalty commitment is the number that maximizes the cost of the commitment to the buyer (and, hence, to the incumbent).

[20]The profit-maximizng monopoly price if $q = 1 - p$ is $(1 + c)/2$.

negative for fewer than five buyers, but because this is a region in which it is not profitable to offer the discount, this region should be ignored.

# 5 Loyalty Discounts Without Buyer Commitment

Our model for loyalty discounts without buyer commitment is identical to our model with commitment except for two features. First, we allow $I$ to make loyalty discount offers sequentially to buyers in period 1. With simultaneous offers, there will always be an equilibriumin which all buyers would refuse to be covered by a loyalty discount. Second, because the offers are sequential, we allow $I$ to only make loyalty discounts available to a subset of buyers. It is usually easy for sellers to offer loyalty discount contracts to some buyers but not others because (1) this form of discrimination does not require ascertaining which buyers value the product more and (2) contracts can prohibit the assignment of contractual rights to others. In the medical industry, suppliers often offer loyalty discount contracts only to members of particular GPOs and thus not to buyers that belong to other GPOs or no GPO at all. We continue to assume that $I$ does not commit to a price at the time it offers a loyalty discount.[21]

In this setting, the pricing equilibrium must be in mixed strategies as long as the discount is large enough that $\pi_e(c) < (1-\theta)\pi_e(c+d)$. To see this, say $E$ chose $p_e > c$. Then $I$ would choose a discounted price to covered buyers ($p_d = p_f - d$) just below $p_e$. $E$'s best response would be either to match it or choose $Min\{p_e^m, c+d\}$ and sell only to uncovered buyers. Because $E$'s best response to a price just below $p_e^m$ would be to match it, there can be no pure strategy equilibrium with $p_e > c$. $\pi_e(c) < (1-\theta)\pi_e(c+d)$, however, guarantees that $p_e \leq c$ is not optimal for $E$.

We first construct an equilibrium in which $d$ is large enough that $I$ does not sell to uncovered buyers with positive probability. The proof of the next proposition shows this equilibrium is the more profitable than any equilibrium with a smaller $d$. In this large $d$

[21] If $I$ were to choose a price along with a loyalty discount (effectively choosing a price to uncovered buyers as well), then $I$ would effectively be a Stackelberg-leader. We examined this case in a prior version of the paper. The results were similar to the simultaneous pricing model we analyze below except that $I$ always chose a price at the bottom its price distribution and $E$ always chose its monopoly price.

equilibrium, $E$ can sell to uncovered buyers only at $p_e^m$.[22] Its price to uncovered buyers is between $[p_0, p_e^m]$ where $p_0$ is given by $\pi_e(p_0) = (1-\theta)\pi_e(p_e^m)$. $E$'s profit equals its monopoly profit from only selling to uncovered buyers. Similarly, $I$'s discounted price ($p_d$, the price covered buyers pay after receiving the discount) is between $[p_0, p_e^m)$. $I$'s distribution function $F$ ensures that $E$ earns $(1-\theta)\pi_e(p_e^m)$ from any price between $[p_0, p_e^m]$. $E$'s pricing distribution function, $F_e$, keeps $I$'s profit constant at $\pi_I(p_0)$ for any price between $[p_0, p_e^m)$.

The following proposition describes the most important features of the equilibrium.

**Proposition 3** *If there exists a $\theta$ such that $s(c) - \int_{p_0}^{p_e^m} s(p)dF(p) \leq \theta N \pi_I(p_0)$, where $p_0$ is given by $\pi_e(p_0) = (1-\theta)\pi_e(p_e^m)$, then the equilibrium if $I$ can offer a loyalty discount without commitment is as follows.*

**A.** *The discount is $d > p_e^m - c$;*

**B.** *The loyalty discount is offered to a fraction of buyers $\theta \leq 1/2$. $\theta$ is increasing in $c_e$ and is $1/2$ at $c_e = c$.*

**C.** *$I$ and $E$'s pricing distributions are given by $F(p) = \frac{\pi_e(p)-(1-\theta)\pi_e(p_e^m)}{\theta\pi_e(p)}$ and $F_e(p) = 1 - \frac{\pi_I(p_0)}{\pi_I(p)}$, respectively where $E$ has an atom at $p_e^m$ of $\frac{\pi_I(p_0)}{\pi_I(p_e^m)}$. Both covered and uncovered buyers will pay a price between $p_0$ and $p_e^m$, strictly exceeding the competitive price.*

**D.** *All offered buyers will agree to be covered for no compensation;*

Proof. See Appendix.

This proposition establishes that $I$ can use loyalty discounts without buyer commitment to prevent $E$ from capturing the entire market, leading both to production inefficiency and prices elevated above the competitive level along with the associated allocation inefficiency. While loyalty discounts without buyer commitment cannot exclude the entrant from the entire market, they ensure that the incumbent retains positive market share (despite its relative inefficiency) and maintains supra-competitive prices.

[22] It is always optimal for $d$ to be large enough that $E$ can sell at $p_e^m$ to uncovered buyers with probability one since this maximizes $E$'s minimum price which increases $I$'s profit.

Some might find it troubling that the equilibrium requires that both $I$ and $E$ play a mixed pricing strategy. It should be noted, however, that there is empirical support for mixed strategy equilibria when firms operate in partially segmented markets. For example, Villas-Boas (1995) finds strong support for the Varian (1980) mixed strategy equilibrium in a model with shoppers and non-shoppers that is not too different from ours. Furthermore, despite the mixed strategy equilibrium, our model does have the clear prediction that the covered buyers always take advantage of their loyalty discount while the entrant has an advantage in selling to buyers who do not receive the discount.

# 6 Conclusion

This article has shown that loyalty discounts with or without buyer commitment can increase prices and, with buyer commitment, sometimes completely exclude a more efficient entrant. Strikingly, this result is possible under any weakly concave demand curve, without any entry costs or economies of scale, even if the buyers are final consumers (or otherwise have independent demand), and with loyalty discounts that are above cost and cover less than half the market. We prove these results without assuming buyer switching costs, financial constraints, limited rival expandability, or any intraproduct bundle of contestable and incontestable demand. Although prior literature suggests that loyalty discounts can have additional anticompetitive effects when those conditions exist, this paper proves that loyalty discounts can have important anticompetitive effects even without those market conditions. Unless the entrant cost advantage is sufficiently large, this equilibrium with anticompetitive effects always occurs for a sufficient number of buyers. Our numerical analysis shows that the scope for this inefficient exclusion with buyer commitment is substantial.

These results disprove the intuitions that above-cost loyalty discounts presumptively reduce prices and cannot exclude an equally efficient rival. These results also disprove the claim that voluntary buyer agreement means the agreement must enhance the buyer welfare. Because the existence of loyalty discounts changes the strategic game between the buyer and the seller, we show that loyalty discounts encourage incumbents to raise prices above but-for levels for both buyers that receive the loyalty discount and those that do not.

Our results have important implications for competition policy. First, loyalty discounts can be anticompetitive even in situations in which ordinary exclusive dealing might not be of concern (such as homogenous markets that have no economies of scale or switching costs and involve sales to final consumers). Second, loyalty discounts can be anticompetitive even when they are above cost. Indeed, raising loyal prices above but-for levels (and thus well above cost) is precisely the anticompetitive effect. Third, loyalty discounts can have anticompetitive effects even when they cover a minority of the market. Fourth, loyalty discounts with buyer commitment can be anticompetitive even if buyer coordination is likely.

A few limitations should be stressed. First, we have assumed a market with only one potential rival. This can often be the case, especially in high-tech or pharmaceutical markets, but if there were multiple rivals with similar costs it is possible they would compete prices down to their costs. However, the same sort of analysis seems likely to apply to the extent only one entrant has a potential cost advantage and the other rivals have higher costs and merely provide a competitive fringe. Multiple rivals might also use loyalty discounts themselves, creating similar or even exacerbated market segmentation effects.[23] It remains for future work to extend the model here to cases involving multiple rivals or where multiple firms use loyalty discounts.[24]

Second, our analysis of loyalty discounts with buyer commitments assumes those commitments are honored. This analysis would get more complicated if we considered the possibility that buyers could breach their commitments. However, as discussed above, if buyers must pay expectation damages when they breach their loyalty commitments, then this would often deter breach.[25] Even if it would not, that would not eliminate the ability of the incumbent to profitably use loyalty discount contracts to reduce competition. Moreover, as Elhauge (2009) has pointed out, there are many reasons why legal or extralegal penalties for breach might exceed expectation damages.

Lastly, our model assumes loyalty discounts have no efficiencies. Efficiencies could offset adverse effects. The model can be extended to show how much of a cost reduction would be

[23]See Elhauge (2009) at pp. 195, 214-15 for a discussion of these issues.

[24]See Kitamura (2010) for an argument that naked exclusion is less effective if there are multiple entrants.

[25]Elhauge (2009) finds that, under linear demand, loyalty commitments will induce rivals to price sufficiently above cost that expectation damages will deter breach.

necessary to offset the anticompetitive effects, but that remains a matter for future work.

# 7 Appendix

**Proof of Proposition 1.** A. (i) $F_e(p) = 1 - \theta\frac{\pi_I^m - \pi_I(p-d)}{(1-\theta)\pi_I(p)}$for $p \in [p_0, p_e^m)$ and $F_e(p) = 0$ for $p < p_0$. From (5), we can see that as $\theta \to 0$, $p_0 \to c$ and as $\theta \to 0$, $F_e(p) \to 1$ for all $p \geq p_0$. This implies that as $\theta \to 0, \Pr[p_e = c] \to 1$. (ii) Since $F_e(p) = 1 - \theta\frac{\pi_I^m - \pi_I(p-d)}{(1-\theta)\pi_I(p)}$for $p \in [p_0, p_e^m)$, we can see that as $\theta \to \bar{\theta}$, $F_e(p) \to 0$ for $p < p_e^m$, which implies that $\Pr[p_e = p_e^m] \to 1$. The definition of $\bar{\theta}$ implies that as $\theta \to \bar{\theta}$, $p_0 \to p_e^m$. Since $F(p) = 1 - \frac{\pi_e(p_0)}{\pi_e(p)}$ for $p_0 \leq p < p_e^m$, this implies that as $\theta \to \bar{\theta}, F(p) \to 0$ for $p_0 \leq p < p_e^m$. From (9), this requires that $\Pr[p_f = p^m + d] \to 1$ as $\theta \to \bar{\theta}$. (iii) $\theta$ only affects $I$'s pricing through its effect on $p_0$. Totally differentiating (5) with respect to $\theta$ and solving for $\frac{dp_0}{d\theta}$ gives $\frac{dp_0}{d\theta} = \frac{\pi_I(p_0) + \pi_I^m - \pi_I(p_0-d)}{(1-\theta)[q(p_0)+(p_0-c)q'(p_0)] + \theta[q(p_0-d)+(p_0-d-c)q'(p_0-d)]} > 0$. So, increasing $\theta$ increases $p_0$ which reduces $F(p)$ for any $p < p_e^m$. Differentiating (6) with respect to $\theta$ gives $-\theta\frac{\pi_I^m - \pi_I(p-d)}{(1-\theta)(p-c)\pi_I(p)} < 0$. So, increasing $\theta$ reduces the probability that $E$ chooses a price below any given level $p < p_e^m$.

B. (i) Inspection of (4) makes it clear that it does not depend on $c_e$. Because increasing $c_e$ increases $p_e^m$, however, the atom in $E$'s distribution at $p_e^m$ is now spread out between $[p_e^m, p_e'^m]$. $p_0$ is also not affected by $c_e$ (see (5)). Differentiating (8) with respect to $c_e$ gives $\frac{(p-p_0)q(p_0)}{(p-c_e)^2 q(p)} > 0$. Thus, for any $p < p_e^m$, the probability that $I$ charges a price less than $p$ is increasing in $c_e$ for $p < p_e^m$. There is a new atom at $p_e'^m$ and positive probability up to $p_e'^m$. (ii) Differentiating (6) with respect to $c$ gives $-\theta\frac{(p^m-p)q(p^m)+dq(p-d)+(dp^m/dc)[q(p^m)+(p^m-c)q'(p^m)]}{(1-\theta)(p-c)^2 q(p)} < 0$. This is negative because the term in square brackets in the numerator is zero because $p^m$ maximizes profits and $p^m > p$ and $d > 0$. $c$ only affects $I$'s pricing distribution, (9), through its effect on $p_0$. Because $p_0 < p^m$, increasing $p_0$ decreases $F(p)$ for any $p < p_e^m$. Totally differentiating (5) with respect to $c$ and solving for $\frac{dp_0}{dc}$ gives $\frac{dp_0}{dc} = \frac{(1-\theta)q(p_0)+\theta(q(p_0-d)-q(p^m))+\theta(dp^m/dc)[q(p^m)+(p^m-c)q'(p^m)]}{(1-\theta)[q(p_0)+(p_0-c)q'(p_0)]+\theta[q(p_0-d)+(p_0-d-c)q'(p_0-d)]} > 0$. This is positive because $p_0 < p^m$, $d > 0$, and $q(p^m) + (p^m - c)q'(p^m) = 0$. So, increasing $c$ increases $p_0$ which reduces $F(p)$ for any $p < p_e^m$. (iii) $d$ only affects $I$'s pricing through $p_0$. Totally differentiating (5) with respect to $d$ and solving for $\frac{dp_0}{dd}$ gives $\frac{dp_0}{dd} = \frac{\theta[q(p_0-d)+(p_0-d-c)q'(p_0-d)]}{(1-\theta)[q(p_0)+(p_0-c)q'(p_0)]+\theta[q(p_0-d)+(p_0-d-c)q'(p_0-d)]} > 0$. So, increasing $d$ increases $p_0$ which reduces $F(p)$ for any $p < p_e^m$. Differentiating (6) with respect to $d$ gives $-\theta\frac{q(p-d)+(p-d-c)q'(p-d)}{(1-\theta)(p-c)q(p)} < 0$. So, increasing $d$ reduces the probability that $E$ chooses a price below any given level $p < p_e^m$. Q.E.D.

**Proof of Remark 1**

If all buyers reject the loyalty discount contract, then all buyers buy at $c$. If one buyer accepts the loyalty discount contract, then it pays a price of $p-d$ where $p$ is distributed according to (9). The consumer surplus associated with any given price is clearly decreasing in price. (i) Expected consumer surplus from signing a loyalty discount contract is $\int_{p_0}^{p_e^m} s(p-d)f(p)dp + \frac{\pi_e(p_0)}{\pi_e^m}s(p^m)$. Differentiating this with respect to $c_e$ (and realizing that $p_e^m$ depends on $c_e$) gives the following:

$$\int_{p_0}^{p_e^m} s(p-d)\frac{df(p)}{dc_e}dp + f(p_e^m)s(p_e^m-d)\frac{dp_e^m}{dc_e} - \frac{(p_e^m-p_0)q(p_0)}{(p_e^m-c_e)^2 q(p_e^m)}s(p^m) \qquad (13)$$

The last term does not contain a $\frac{dp_e^m}{dc_e}$ term because the term multiplying it is zero by the envelope theorem. Differentiating (8) with respect to $p$ and evaluating at $p_e^m$ gives $f(p_e^m) = \frac{\pi_e(p_0)(q(p_e^m)+(p_e^m-c_e)q'(p_e^m))}{\pi_e(p_0)^2} = 0$ because $q(p_e^m)+(p_e^m-c_e)q'(p_e^m) = 0$ by definition of $p_e^m$. Because surplus is decreasing in price, the first term is greater than $s(p_e^m-d)\frac{dF(p_e^m)}{dc_e}$ where $F(p_e^m)$ is given by (8) rather than (9). Thus, the effect of $c_e$ on expected surplus from signing is greater than $\frac{(p_e^m-p_0)q(p_0)}{(p_e^m-c_e)^2 q(p_e^m)}(s(p_e^m-d)-s(p^m)) > 0$ (because $\frac{dF(p_e^m)}{dc_e} = \frac{(p_e^m-p_0)q(p_0)}{(p_e^m-c_e)^2 q(p_e^m)}$). The consumer surplus from rejecting is clearly independent of $c_e$ because the price is fixed at $c$. Thus, the expected loss in consumer surplus from signing the loyalty discount contract is decreasing in $c_e$. Thus, the amount necessary to compensate one buyer for signing the loyalty discount is decreasing in $c_e$. Because $I$ plays a mixed pricing strategy which includes in its support charging the monopoly price to all signing buyers, its profit is simply the monopoly profit per buyer times the number of signing buyers. Thus, $I$ will pay up to the monopoly profit per buyer to induce buyers to sign. So, if the amount necessary to compensate one buyer for signing the loyalty discount contract is less than the monopoly profit per buyer at $c_e$, then it is less for any $c_e > c_e$. (ii) Proposition 2A says the amount necessary to compensate one buyer for signing the loyalty discount is decreasing in $N$. So, if the amount necessary to compensate one buyer for signing the loyalty discount contract is less than the monopoly profit per buyer at $\bar{N}$, then it is less for any $N > \bar{N}$. Q.E.D.

**Proof of Proposition 3**

We first characterize the large $d$ equilibrium by proving that the distribution functions in the proposition constitute an equilibrium and establish the formula for the $\theta$ that maximizes $I$'s profits in this equilibrium. This proofs all parts of the proposition except part A (the size of $d$). Lastly, we prove (in a web appendix) that any equilibrium in which $I$ sells to uncovered buyers with positive probability leads to lower profits (this proves part A).

## Characterzing the large $d$ equilibrium

First, we prove C. We obtain $E$'s distribution function $F_e$ as follows. $I$'s expected profit from a discounted price of $p_d = p_f - d$ is $\theta(1 - F_e(p_d))\pi_I(p_d)$. $E$'s maximum price is $p_e^m$. $E$'s minimum price is $p_0$ where $p_0$ solves $\pi_e(p_0) = (1-\theta)(p_e^m - c_e)q(p_e^m)$ because $E$ can always earn $(1-\theta)\pi_e(p_e^m)$ from selling only to uncovered buyers.[26] Thus, $F_e$ must be such that $I$'s profit from charging $p_d = p_0$ and selling to covered buyers with probability one equals its profit from charging $p_d$ and selling to covered buyers with probability $1 - F_e(p_d)$. That is:

$$\theta(1 - F_e(p_d))\pi_I(p_d) = \theta\pi_I(p_0)$$

This implies that

$$F_e(p_d) = 1 - \frac{\pi_I(p_0)}{\pi_I(p_d)} \tag{14}$$

with an atom at $p_e^m$ of $\frac{\pi_I(p_0)}{\pi_I(p_e^m)}$.

To obtain $I$'s pricing distribution,$F$, note that $E$'s expected profit from charging $p_e$ is $\theta(1 - F(p_e))\pi_e(p_e) + (1-\theta)\pi_e(p_e)$. $E$ sells to uncovered buyers with probability one and covered buyers with probability $1 - F(p_e)$. If $E$ charges $p_e^m$ and sells only to uncovered buyers, its profit is $(1-\theta)\pi_e(p_e^m)$.[27] So, mixing requires that:

$$\theta(1 - F(p_e))\pi_e(p_e) + (1-\theta)\pi_e(p_e) = (1-\theta)\pi_e(p_e^m)$$

[26]Since $p_0 \geq c$, this creates a maximum $\theta$ for the mixed strategy equilibrium. For any $\theta$ above this maximum, the entrant will charge $c$ with probability one. Obviously, any $\theta$ above this maximum will not be in the interest of the incumbent, since it will earn zero profits. The maximum $\theta = 1 - \frac{(c-c_e)q(c)}{(p_e^m - c_e)q(p_e^m)}$.

[27]In equilibrium,the incumbent's maximum price cannot exceed the entrant's maximum price or the incumbent would make no profits. Thus, the entrant only sells to uncovered buyers at its maximum price. Given this, the entrant's maximum price must be $p_e^m$.

This implies that

$$F(p_e) = \frac{\pi_e(p_e) - (1-\theta)\pi_e(p_e^m)}{\theta\pi_e(p_e)} \tag{15}$$

Now, we prove part B. That is, we determine the fraction of covered buyers that maximizes $I$'s profits given the pricing distributions. $I$'s expected profit is $\theta(p_0 - c)q(p_0)$. The condition for $p_0$ implies that $dp_0/d\theta = -(p_e^m - c_e)q(p_e^m)/\{q(p_0) + (p_0 - c_e)q'(p_0)\}$; using this, the first order condition for $\theta$ is:

$$\theta = \frac{\pi_I(p_0)\{q(p_0) + (p_0 - c_e)q'(p_0)\}}{\pi_e(p_e^m)\{q(p_0) + (p_0 - c)q'(p_0)\}} \tag{16}$$

The right hand side of (16) is increasing in $c_e$. At $c = c_e$, we get $\theta = \pi_I(p_0)/\pi_I(p_e^m) = 1 - \theta$, where the last equality follows from the implicit condition for $p_0$. This means that at $c = c_e$, $I$ maximizes period 2 profits at $\theta = 1/2$. If $E$ has a cost advantage, then $I$ will cover a smaller fraction of the market.

We now show D, that given this pricing equilibrium, $I$ can cover $\theta$ buyers if $s(c) - \int_{p_0}^{p_e^m} s(p)dF(p) \leq \theta N \pi_I(p_0)$ for zero compensation, making the above $\theta$ optimal. Notice that $p_{0,}$ the minimum of both price distributions, is decreasing in $\theta$; $F_e(p_d)$ is decreasing in $p_0$ (and the atom at $p_e^m$ is increasing in $p_0$); and $F(p_e)$ is increasing in $\theta$. This means that buyers fact lower prices (in expectation) from both $I$ and $E$ as $\theta$ increases (for $\theta > 0$). Thus, once one buyer is covered, other buyers strictly prefer to be covered than uncovered. Hence, once one buyer is covered, all other buyers will agree to be covered. If $s(c) - \int_{p_0}^{p_e^m} s(p)dF(p) \leq \theta N \pi_I(p_0)$, then $I$ can profitably compensate one buyer for its loss in consumer surplus. As long as this condition holds, however, then $I$ can get $\theta N$ to agree without compensation since as soon as all buyers but $\theta N$ have rejected, $I$ will offer this compensation. Anticipating that, earlier buyers will to be covered for no compensation.

# 8 Web Appendix

### Linear Demand Numerical Analysis

Solving (5) explicitly for $p_0$ gives $p_0(\theta) = \{(1 + c + 2d\theta) - \sqrt{(1-\theta)((1-c)^2 - 4d\theta)}\}/2$.

This gives closed form expressions for both $F$ and $F_e$. Furthermore, the assumption of linear demand gives a closed form expression for consumer surplus for any given price, and for $I$ and $E$'s monopoly price. This means we can write expected consumer surplus for both committed and free buyers as a function of the number of committed buyers, $n$. If $n/N \geq \bar{\theta}$, then the consumer surplus for free buyers is simply $s(p_e^m)$ and the consumer surplus for committed buyers is $s(p^m)$ since both firms charge their monopoly profits for $\theta \geq \bar{\theta}$. In this case, our assumption that $(p^m - c)q(p^m) > s(p_e^m) - s(p^m)$ ensures that exclusion is profitable.

Since we want to examine whether exclusion is profitable for any possible set of buyer beliefs about that the number of other buyers that will sign a loyalty discount contract, we also need to examine expected consumer surplus for free and committed buyers if $n/N < \bar{\theta}$. In this case, expected consumer surplus depends on the prices from the mixed strategy equilibrium. The expected consumer surplus for both committed and free buyers as a function of the number of committed buyers, $n$, for $n/N < \bar{\theta}$ is given by:

$$\begin{aligned} Es^c(n) &= \int_{p_0(n)}^{(1+c_e)/2} \frac{1}{2}(1-p-d)^2 \frac{(1+c_e-2p)(1-p_0(n))(p_0(n)-c_e)}{(1-p)^2(p-c_e)^2} dp + \\ & \frac{4(1-p_0(n))(p_0(n)-c_e)}{(1-c_e)^2}(1-c)^2/8 \end{aligned} \tag{17}$$

$$\begin{aligned} Es^f(n) &= \int_{p_0(n)}^{(1+c_e)/2} \frac{1}{2}(1-p)^2 \frac{(n/N)(1+c-2(p-d))^2}{4(1-p)(p-c)(1-(n/N))} \frac{(1+c_e-2p)(1-p_0(n))(p_0(n)-c_e)}{(1-p)^2(p-c_e)^2} dp + \\ & \int_{p_0(n)}^{\frac{(1+c_e)}{2}} \frac{1}{2}(1-p)^2 \frac{(1-p_0(n))(p_0(n)-c_e)}{(1-p)(p-c_e)} \frac{n(1+c-2(p-d))((1-c)^2+2d(1+c-2p))}{4N(1-(n/N))(1-p)^2(p-c)^2} dp + \\ & \frac{4(1-p_0(n))(p_0(n)-c_e)}{(1-c_e)^2} \frac{(n/N)(c-c_e+2d)}{(1-(n/N))(1-c_e)(1-2c+c_e)} (1-c_e)^2/8 \end{aligned} \tag{18}$$

In these expressions, $p_0(n)$ is simply $p_0(\theta)$ with $\theta = n/N$. The first expression is the expected surplus for committed buyers when $n$ buyers will be committed. The integral term is the expected surplus when $I$ prices between $p_0$ and $p_e^m$. For any given price, a committed buyer's surplus is $\frac{1}{2}(1-p-d)^2$ (because she gets the discount of $d$ off the price of $p$ offered to free buyers). $I$'s probability density function in this region is $\frac{(1+c_e-2p)(1-p_0(n))(p_0(n)-c_e)}{(1-p)^2(p-c_e)^2}$. This can be computed simply by differentiating $I$'s cumulative distribution function, equation (9), under the linear demand assumption. $I$ will also charge committed buyers the monopoly price with probability $\frac{4(1-p_0(n))(p_0(n)-c_e)}{(1-c_e)^2}$ and when that happens these buyers obtain a surplus of $(1-c)^2/8$.

The second expression is the surplus for free buyers. This contains three terms. The first two reflect the expected surplus when neither $I$ nor $E$ charge their monopoly prices. The first is the expected surplus the buyer gets when buying from $I$, which occurs when $E$'s price is higher. For any given incumbent price $p$, the probability of this is $\frac{(n/N)(1+c-2(p-d))^2}{4(1-p)(p-c)(1-(n/N))}$. This is simply $1-F_e(p)$ with linear demand. The second term is the expected surplus the buyer gets when buying from $E$, which occurs when $I$'s price is higher. For any given entrant price $p$, the probability of this is $\frac{(1-p_0(n))(p_0(n)-c_e)}{(1-p)(p-c_e)}$. Again, this is $1-F(p)$ with linear demand. The last part of this term is entrant's probability density function in this region with linear demand. The last term is the expected surplus if both $I$ and $E$ charge their monopoly prices. The first expressions in this last line are the probability that this occurs, the $(1-c_e)^2/8$ expression is the buyer's surplus in this case (because the buyer buys from $E$ because $c_e \leq c$).

Unfortunately, it is not possible to get closed-form solutions for these integrals for arbitrary $c_e$ and $d$. We proceed by numerical analysis. For any given set of parameter values, we numerically calculate the consumer surplus functions to determine the loss in expected consumer surplus a buyer would receive from signing a loyalty discount contract with any given discount $d$ given that exactly $n$ other buyers will sign the contract. This gives the maximum amount $I$ must pay to induce $n+1$ buyers to sign a loyalty discount contract with discount $d$. We call this the signing cost for $n$ signers. (It is the maximum amount because it may be cheaper to induce $n'+1$ buyers to sign the contract for some $n' > n$. Thus, if buyers expect $n'$ other buyers to sign the contract, $I$ may not need to pay as much.) We then numerically calculate the number of buyers, $n$, out of the total number of buyers, $N$, for which this signing cost reaches its maximum. This gives the maximum amount $I$ would have to pay to induce all buyers to sign the loyalty discount contracts.[28] We call this the maximum signing cost for any given discount, $d$. We then find, numerically, the discount,

[28]This is the maximum because then every buyer will sign for any belief about how many other buyers will sign.

$d$, that minimizes this maximum signing cost.

This procedure allows us to determine, for any given $c, c_e$, and $N$, the discount, $d$, that allows the firm to induce all buyers to sign a loyalty discount contract (given any beliefs about what other buyers will do) for the minimum up-front transfer, $t$. (The next subsection focuses on the optimal $d$.) We can further determine how large this minimum $t$ has to be (this is simply the value of the maximum signing cost at the $d$ that minimizes this value) and whether it is less than the firm's profit from inducing the buyer to sign. This profit is simply $(1-c)^2/4$, the monopoly profit per buyer. Using numerical root finding, we can also determine the minimum value of $c_e$ (for any given $c$ and $N$) for which the firm can induce all buyers to sign a loyalty discount contract in any equilibrium.[29]

We use this procedure to find this minimum value for $c_e$ for every integer value of $N$ between 3 and 20 (for $N = 2$ we observed that it is not possible to induce one buyer to sign a loyalty discount if she believes the other buyer will not sign even if $c_e = c$) and for $c \in \{.1, .2, ..., .9\}$. Using this data, we then run a regression to generate a functional form for finding this minimum value for $c_e$ as a function of $N$ and $c$, expression (11) in the text.

Our numerical analysis to determine the optimal size of the loyalty discount proceeded as follows. We randomly drew parameter values for $I$'s cost, $c$, $E$'s cost, $c_e$, and the number of buyers, $N$. We drew $c$ from a uniform distribution between zero and one. We drew $c_e$ from a uniform distribution between $Max\{0, 2c-1\}$ and $c$. This ensures both that $E$'s innovation is non-drastic and that it has a cost advantage over $I$.[30] We drew $N$ from the integers between three and 20 with each one equally likely. For each set of parameters that we drew, we calculated, as above, the discount, $d$, that minimized $I$'s maximum cost of signing each buyer, assuming that the each buyer assumed that the number of other buyers who would sign the loyalty discount contract was the number that would maximize the cost of inducing

[29]While Proposition 3 tells us that the cost of obtaining the first signer is monotonically decreasing in $c_e$, we have not proved analytically that the cost of obtaining any number of signers is monotonically decreasing in $c_e$. That said, numerical analysis for the linear case has revealed no instances where the minimum (over $d$) of the maximum cost of inducing all buyers to sign is increasing in $c_e$. Furthermore, as we discuss in more detail below, except when the number of buyers is very large, this analysis also shows that the maximum cost occurs when buyers expect no other buyers to sign. This is precisely the case where Proposition 3 proves that the cost is monotonically decreasing in $c_e$.

[30]This restriction on the entrant's cost is actually somewhat weaker than Assumption $(*)$, which explains why loyalty discounts were not profitable in so many cases.

buyers to sign the contract.[31] Thus, for each set of parameter values, we determined the optimal discount and the cost of signing up buyers under the assumption that each buyer's belief about the number of other buyers who would sign the contract is that which would maximize the signing cost.

We did 1500 draws from these distributions of parameter values. Of these 1500 draws, using loyalty discounts was profitable (and effectively excluded $E$) in just over half of the draws (763).[32] We used these 763 cases to analyze (using linear regression analysis) the optimal loyalty discount in situations where $I$ would use them. We found that a relatively simple, and easy to interpret specification, explained over 90 percent of the variation in the optimal discount (R-squared of .901). This is expression (12) in the text.

**Proposition 3 Proof that Smaller $d$ equilibria are less profitable**

Now we show part A, that this large $d$ equilibrium is the most profitable one for $I$. First, we characterize three possible equilibria for three different levels of $d$ and the conditions on $d$ for each one. Then we show that the large $d$ equilibrium is the most profitable.

To characterize the large $d$ equilibrium in the text, note that this is an equilibrium if the incumbent does not want to deviate by choosing a discounted price of $p_e^m - d$ or smaller in order to sell to uncovered buyers with positive probability. If $p_e^m - d \leq p_0$, then the incumbent sells to to covered buyers with probability one. Because $F_e(p)$ is defined by the equation $(1 - F_e(p))\pi_I(p) = \pi_I(p_0)$, the incumbent's profit from any prices between $p_0$ and $p_e^m$ is constant. So, if its price to uncovered buyers is $p_f \in [p_0, p_e^m]$ and its price to covered buyers is $p_f - d \leq p_0$, then its total profit from this deviation will be greatest at $p_f = p_e^m$ since this maximizes its profit from covered buyers. This deviation is not profitable if and only if:

$$\theta\pi_I(p_0) \geq \theta\pi_I(p_e^m - d) + (1 - \theta)(1 - F_e(p_e^m))\pi_I(p_e^m)$$

Using $(1 - F_e(p))\pi_I(p_d) = \pi_I(p_0)$, we can rewrite this as:

[31] We make this assumption to examine the worst case for loyalty discounts for the incumbent.

[32] Not surprisingly, given the results in the last subsection, the cases where loyalty discounts were not profitable were those where the entrant's cost advantage was large and the number of buyers was small, and the market was small.

$$(2\theta - 1)\pi_I(p_0) - \theta\pi_I(p_e^m - d) \geq 0 \quad (19)$$

Because $\theta \leq 1/2$, a necessary condition for this deviation to not be profitable is $d \geq p_e^m - c$. Because $p_0 > c$, this implies that $p_e^m - d < p_0$. So, the large $d$ equilibrium occurs if $(2\theta - 1)\pi_I(p_0) - \theta\pi_I(p_e^m - d) \geq 0$. This condition will clearly hold if $d$ is large enough for any given $\theta > 0$ as long as there no restriction on the maximum size of $d$.

The small $d$ equilibrium occurs if $\pi_e(c) \geq (1 - \theta)\pi_e(c + d)$ and $c + d < p_e^m$. If this condition holds, then there exists a pure strategy equilibrium in which $p_e = p_d = c$ and the entrant captures the entire market. Such an equilibrium is obviously less profitable to the incumbent than the large $d$ equilibrium.

The intermediate $d$ equilibrium occurs when $d$ is large enough that this pure strategy equilibrium does not exist, but small enough that the large $d$ equilibrium does not exist (so $I$ must sell to uncovered buyers with positive probability). (If such a region does not exist, then the proof is complete. So we will assume it does exist.) The argument in the text establishes that this equilibrium must also be in mixed strategies.

To characterize the equilibrium in this case, let the support of $I$'s pricing distribution to covered buyers be $[p^B, p^T]$ (so, the support for uncovered buyers would be $[p^B + d, p^T + d]$) and the support of $E$'s pricing distribution be $[p_e^B, p_e^T]$. Note than in any equilibrium, $p_e^T \geq p^T$ since $I$ would have no sales at $p_d > p_e^T$. Similarly, $p_e^B \geq p^B$ because $p^B \leq p_e^m$ and so for any $p_e < p^B$, $E$'s profit is strictly increasing in $p_e$ because $E$ will be making sales to all buyers with probability one if $p_e < p^B$.

The following lemma provides an important conditon of the equilibrium if $\theta > 1/2$.

**Lemma** *If $\theta > 1/2$ and $I$ sells to uncovered buyers with positive probability, then $p^T - d \leq p^B$.*

Proof. If $p_e^B > p^B$, then $p_e^T - d > p^B$ or else $p^B$ cannot maximize $I$'s profits. For any $p \in [p^B, Min\{p_e^B, p_e^T - d\})$, $I$'s profit is $\theta\pi_I(p) + (1 - \theta)(1 - F_e(p + d))\pi_I(p + d)$. This implies that $F_e(p) = 1 - \frac{K - \theta\pi_I(p-d)}{(1-\theta)\pi_I(p)}$ for $p \in [p^B + d, Min\{p_e^B + d, p_e^T\})$ if $p^T > p^B + d$. Notice, however, that for $p \in [p^B + d, Min\{p_e^B + d, p_e^T\})$, then $I$'s profits are $\theta\frac{K - \theta\pi_I(p-d)}{(1-\theta)} + (1 - \theta)(1 - F_e(p + d))\pi_I(p + d)$. Because $\theta > 1/2$ and $\pi_I$ is concave, however, this is decreasing in $p$

unless $F_e$ is decreasing, which is impossible for a distribution function. Thus, $p^T - d \leq p^B$ if $p_e^B > p^B$. Now consider $p_e^B = p^B$. If $p_e^T > p^T$, then for $p_e \in [p^T, Min\{p_e^T, p^T + d\}]$, $E$'s profit is $(1-\theta)(1-F(p_e - d))\pi_e(p_e) = \pi_e(p^B)$. For $p_e \in [p^T - d, Min\{p_e^T - d, p^T\}]$, $E$'s profit is $\theta(1 - F(p_e))\pi_e(p_e) + (1-\theta)(1 - F(p_e - d))\pi_e(p_e) = \pi_e(p^B)$. Since $F(p_e)$ in this region is $F(p_e - d)$ for $p_e \in [p^T, Min\{p_e^T, p^T + d\}]$, so we can write $E$'s profit in this region as $\theta\pi_e(p^B)/(1-\theta) + (1-\theta)(1-F(p_e - d))\pi_e(p_e) = \pi_e(p^B)$. This implies that $(1-\theta)^2(1-F(p_e-d))\pi_e(p_e) = (1-2\theta)\pi_e(p^B)$. For $\theta > 1/2$, the right hand side is negative while the left hand side is positive. So, this can't be an equilibrium. The remaining case is that of identical support. If $p \in [p^T - d, p^T]$, $I$ only sells to covered buyers, so has a profit of $\theta(1 - F_e(p))\pi_I(p) = \bar{\pi}$. If $p \in [Max\{p^T - 2d, p^B\}, p^T - d]$, $I$'s profit is $\theta(1-F_e(p))\pi_I(p) + (1-\theta)(1-F_e(p+d))\pi_I(p+d) = \bar{\pi}$. Since $p+d \in [p^T - d, p^T]$, we can use the profit expression for $p \in [p^T - d, p^T]$, to write this as $\theta(1-F_e(p))\pi_I(p) + (1-\theta)\bar{\pi}/\theta = \bar{\pi}$. This implies $\theta^2(1-F_e(p))\pi_I(p) = (2\theta - 1)\bar{\pi}$. At $p = p^T - d$, we have that $(1-F_e(p^T-d)) = \frac{\bar{\pi}}{\theta\pi_I(p^T-d)}$ from the right and $(1-F_e(p^T-d)) = \frac{(2\theta-1)}{\theta}\frac{\bar{\pi}}{\theta\pi_I(p^T-d)}$ from the left. At $1 > \theta > 1/2$, however, $\frac{(2\theta-1)}{\theta} < 1$, which means $F_e(p^T - d)$ is smaller from the right than from the left, which is not possible. Q.E.D.

Using this, we can now provide a key characterization of the equilibrium for all $\theta$.

**Lemma** *If $I$ sells to uncovered buyers with positive probability, then $p^T - p^B = d$ and $p_e^T - p_e^B = d$.*

Proof of Lemma. At $p_d$ in the neighborhood of $p^T$, $I$ can only sell to covered buyers, otherwise at $p_e^T$, $E$ would have no sales, which would make this not part of an equilibrium pricing strategy. This implies that $p_e^T - d < p^T$. Thus, if $F_e$ is $E$'s pricing distribution function, $I$'s profit from a price $p_d \in [p_e^T - d, p^T]$ is $\theta(1 - F_e(p_d))\pi_I(p_d)$. Since $I$ is playing a mixed strategy, this profit has to be constant, which implies that in this region $F_e(p) = 1 - \frac{k}{\pi_I(p)}$ and that $I$'s profit from choosing price in this region is $\theta k$. $I$'s profit from $p_d \in [p_e^T - 2d, p^T - d]$ is then $\theta(1 - F_e(p_d))\pi_I(p_d) + (1-\theta)k$. $\theta \leq 1/2$, implies that both of these regions could only be part of $I$'s support if $p^T - d \leq c$. If so, however, then $I$'s profit is not constant over $p_d \in [p_e^T - 2d, p^T - d]$. Thus, $p^T - d \leq p^B$ for $\theta \leq 1/2$; the last lemma proved this for $\theta > 1/2$.

If $I$ does not sell to uncovered buyers with positive probability, then the equilibrium in the text obtains. If it does, then $p^B + d < p_e^T$, which means that $p^T < p_e^T$ . If $p^T - d < p^B$, however, then at $p_e \in [p^T, Max\{\ p^B + d, p_e^m\})$, $E$'s profit is $(1-\theta)\pi_e(p_e)$, which is strictly increasing in $p_e$. Thus, $E$'s mixed strategy requires that $p^T - p^B = d$.

At $p_e \geq p^T$, $E$ only sells to uncovered buyers, so its profit is $(1-\theta)(1-F(p_e-d))\pi_e(p_e)$. Constant profit in this region implies that $F(p) = 1 - \frac{k_e}{\pi_e(p+d)}$ for $p \in [p^T - d, p_e^T - d]$. $E$'s profits from $p_e \geq p^T$ are $(1-\theta)k_e$. At $p_e = p^T$, $E$ sells to uncovered buyers with probability one because $p^T - d \leq p^B$, meaning that its profit is $(1-\theta)\pi_e(p^T)$ which implies that $k_e = \pi_e(p^T)$.

For $p_e < p_e^T - d$, $E$'s profit from covered buyers is $\theta(1-F(p_e))\pi_e(p_e) = \theta\pi_e(p^T)$. Its profit from uncovered buyers is $(1-\theta)\pi_e(p_e)$ because $p_e^T - d < p^T \leq p^B + d$. Since $E$'s profit from covered buyers in this region is constant while its profit from uncovered buyers is not, $p_e^B \geq p_e^T - d$. Notice, however, that if $p_d = p_e^T - d$, then $I$ sells to covered buyers with probability one and uncovered buyers with probability zero. If $p_e^B > p_e^T - d$, however, $I$ could continue to sell to covered buyers with probability one at $p_d > p_e^T - d$, which would mean that $p^B > p_e^T - d$, which would mean that $I$ never sells to uncovered buyers. Thus, $p_e^T - p_e^B = d$ in any equilibrium in which $I$ sells only to covered buyers. Q.E.D.

This lemma means that $p^T \geq p_e^B$, otherwise $p^T + d < p_e^T$, which would mean that $E$ makes zero profit at $p_e^T$, so this price could not be part of its optimal strategy. Thus, $p_d = p_e^B$ is part of $I$'s mixed strategy, meaning that $I$'s expected profit from this equilibrium is $\theta\pi_I(p_e^B)$.

The next lemma characterizes the mixed strategy pricing equilibrium in this case.

**Lemma** *$I$'s pricing distribution is given by $F(p) = 1 - \frac{\pi_e(p^T)}{\pi_e(p+d)}$ for $p \in [p^B, p_e^T - d]$ and $F(p) = \frac{1}{\theta} - \frac{1-\theta}{\theta}\frac{\pi_e(p^T)}{\pi_e(p)}$ for $p \in [p_e^T - d, p^T]$. $E$'s pricing distribution is given by $F_e(p) = 1 - \frac{\pi_I(p_e^B)}{\pi_I(p)}$ for $p \in [p_e^B, p^T]$ and $F_e(p) = 1 - \frac{\theta}{1-\theta}\frac{\pi_I(p_e^B) - \pi_I(p-d)}{\pi_I(p)}$ for $p \in [p^T, p_e^T]$.*

Proof of Lemma. The proof of the last lemma established the first part of $I$'s pricing distribution. For $p_e \in [p_e^B, p^T]$, $E$'s profit from covered buyers is $\theta(1-F(p_e))\pi_e(p_e)$. Its profit from uncovered buyers is $(1-\theta)\pi_e(p_e)$ since $p^T = p^B + d$. Thus, $E$'s profit is constant in this region if and only if $F(p) = \frac{1}{\theta} - \frac{\tilde{k}}{\pi_e(p)}$. This produces expected profit of $\theta\tilde{k}_e$. This profit must equal the profit from $p_e \in [p^T, p_e^T]$, which the proof of the last lemma established was $(1-\theta)\pi_e(p^T)$. Thus, $\tilde{k}_e = (1-\theta)\pi_e(p^T)/\theta$, which produces the desired result.

From the proof of the last lemma, we know $F_e(p) = 1 - \frac{k}{\pi_I(p)}$ for $p \in [p_e^B, p^T]$. Since $I$'s profit at $p_e^B$ is $\theta\pi_I(p_e^B)$ because $I$ sells to covered buyers with probability one and uncovered buyers with probability zero, this implies that $k = \pi_I(p_e^B)$. For $p_d \in [p^T - d, p_e^T - d]$, $I$ sells to covered buyers with probability one and uncovered buyers with probability $1 - F_e(p_d)$, so its total profit is $\theta\pi_I(p_d) + (1-\theta)(1 - F_e(p_d + d))\pi_I(p_d + d)$. Constant profits in this region require that $F_e(p) = 1 - \frac{\tilde{k}}{\pi_I(p)} + \frac{\theta\pi_I(p-d)}{(1-\theta)\pi_I(p)}$ *for* $p \in [p^T, p_e^T]$. This generates profits of $(1-\theta)\tilde{k}_e$. Since this must equal profits from lower pricing, $\tilde{k} = \frac{\theta\pi_I(p_e^B)}{1-\theta}$ which produces the desired result. Q.E.D.

Because the distribution functions are continuous, we know that $\frac{1}{\theta} - \frac{1-\theta}{\theta}\frac{\pi_e(p^T)}{\pi_e(p_e^B)} = 1 - \frac{\pi_e(p^T)}{\pi_e(p_e^T)}$, which means that $\pi_e(p_e^B) = (1-\theta)\pi_e(p_e^T)\frac{\pi_e(p^T)}{\theta\pi_e(p^T) + (1-\theta)\pi_e(p_e^T)}$. Because $\pi_e(p_e^T) > \pi_e(p^T)$ and $p_e^T < p_e^m$, we have that $p_e^B < p_0$. Thus, $I$'s profit from this equilibrium is less than $I$'s profit from the large $d$ equilibrium. Q.E.D.

# References


[1] Aghion, Philippe, and Patrick Bolton. 1987. "Contracts as a Barrier to Entry." American Economic Review, 77(3): 388–401.

[2] Argention, Cedric. 2010. "Exclusive Quality." Journal of Industrial Economics, 58:690-716.

[3] Bernheim, B. Douglas, and Michael D. Whinston. 1998. "Exclusive Dealing." Journal of Political Economy, 106(1): 64–103.

[4] Butz, David A. 1990. "Durable-Good Monopoly and Best-Price Provisions," American Economic Review 80: 1062-76.

[5] Chen, Zhijun and Greg Shaffer. 2010. "Naked Exclusion with Minimum Share Requirements," Working Paper.

[6] Cooper, Thomas E. 1986. "Most-Favored-Customer Pricing and Tacit Collusion," The RAND Journal of Economics 17: 377-388.

[7] Crane, Daniel A. 2013. “Bargaining Over Loyalty,” Texas Law Review, 92: 253-301.

[8] Edlin, Aaron. 1997. ”Do Guaranteed-Low-Price Policies Guarantee High Prices, and Can Antitrust Rise to the Challenge?”, Harvard Law Review 111: 528-575.

[9] Elhauge, Einer. 2002. “The Exclusion Of Competition For Hospital Sales Through Group Purchasing Organizations.” Report to U.S. Senate (June 25, 2002).

[10] Elhauge, Einer. 2003. “Defining Better Monopolization Standards.” Stanford Law Review 56: 253-344.

[11] Elhauge, Einer. 2008. United States Antitrust Law & Economics. Foundation Press: New York.

[12] Elhauge, Einer. 2009. “How Loyalty Discounts Can Perversely Discourage Discounting,” Journal of Competition Law & Economics 5: 189-231.

[13] Elhauge, Einer and Abraham L. Wickelgren. 2013. “Anti-Competitive Market Division Through Loyalty Discounts Without Buyer Commitment,” Mimeo.

[14] Eliat, Assaf, Jirth Jayaratne, Janusz A. Ordover, and Greg Shaffer. 2010. ”How Loyalty Discounts Can Perversely Discourage Discounting: Comment.” The CPI Antitrust Journal April 2010(1).

[15] Farrell, Joseph. 2005. “Deconstructing Chicago on Exclusive Dealing,” Antitrust Bulletin 50: 465-480 (2005).

[16] Fumagalli, Chiara, and Massimo Motta. 2006. “Exclusive Dealing and Entry, when Buyers Compete.” American Economic Review, 96(3):785–95.

[17] Hovenkamp, Herbert. 2005. Antitrust Law Vol.. XI (2d ed.).

[18] Hovenkamp, Herbert. 2006. “Discounts and Exclusion.” Utah Law Review, 2006: 841-861.

[19] Hviid, Morten and Greg Shaffer. 2010. ”Matching Own Prices, Rivals’ Prices or Both?” Journal of Industrial Economics, 58:479–506.

[20] Innes, Robert, and Richard J. Sexton. 1994. "Strategic Buyers and Exclusionary Contracts." American Economic Review, 84(3): 566–84.

[21] Kaplow, Louis & Carl Shapiro. 2007. "Antitrust," in Handbook of Law & Economics 2: 1073-1225 (eds. Polinsky & Shavell).

[22] Kitamura, Hiroshi. 2010. "Exclusionary Vertical Contracts with Multiple Entrants." International Journal of Industrial Organization, 28:213-219.

[23] Kolay, Sreya, Greg Shaffer, and Janusz A. Ordover. 2004. "All-Units Discounts in Retail Contracts," Journal of Economics and Management Strategy, 13:429-459.

[24] Lambert, Thomas A.. 2005. "Evaluating Bundled Discounts." Minnesota Law Review, 89: 1688-1757.

[25] Lambert, Thom. 2012. "Have Elhauge and Wickelgren Undermined the Rule of Per Se Legality for Above-Cost Loyalty Discounts?". Available at http://truthonthemarket.com/2012/09/12/have-elhauge-and-wickelgren-undermined-the-rule-of-per-se-legality-for-above-cost-loyalty-discounts/.

[26] Lin, Y. Joseph. 1990. "The Dampening of Competition Effect of Exclusive Dealing," Journal of Industrial Economics, 39:209-223.

[27] Marx, Leslie M. & Greg Shaffer. 2004. "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity: Comment," American Economic Review 94: 796-801.

[28] Marx, Leslie M. & Greg Shaffer. 2004. "Rent-Shifting, Exclusion, and Market-Share Discounts," Working Paper.

[29] Mathewson, G. Frank, and Ralph A. Winter. 1987. "The Competitive Effects of Vertical Agreements: Comment." American Economic Review, 77(5): 1057–62.

[30] Neeman, Zvika. 1999. "The Freedom to Contract and the Free-Rider Problem." Journal of Law, Economics, and Organization, 15(3): 685–703.

[31] Ordover, Janusz and Greg Shaffer. 2007. "Exclusionary Discounts," Working Paper.

[32] Rasmusen, Eric B., J. Mark Ramseyer, J., and John S, Wiley. 1991. "Naked Exclusion", American Economic Review, 81(5):1137–45.

[33] Rey, Patrick and Joseph E. Stiglitz. 1995. "The Role of Exclusive Territories in Producers' Competition," The RAND Journal of Economics 26: 431-451.

[34] Schnitzer, Monika. 1994. "Dynamic duopoly with best-price clauses," The RAND Journal of Economics 25: 186-196.

[35] Segal, Ilya R., and Michael Whinston. 2000. "Naked Exclusion: Comment." American Economic Review, 90: 296-309.

[36] Simpson, John and Abraham L. Wickelgren. 2007. "Naked Exclusion, Efficient Breach, and Downstream Competition." American Economic Review, 97: 1305-1320.

[37] Spector, David. 2005. "Loyalty Rebates: An Assessment of Competition Concerns and a Proposed Structured Rule of Reason," Competition Policy International 1(2): 89-114 (Autumn).

[38] Tom, Willard K., David A. Balto & Neil W. Averitt. 2000. "Anticompetitive Aspects of Market-Share Discounts and Other Incentives to Exclusive Dealing," Antitrust L.J. 67: 615.

[39] Villas-Boas, J. Miguel. 1995. "Models of Competitive Price Promotions: Some Empirical Evidence from the Coffee and Saltine Crackers Markets," Journal of Economics and Management Strategy, 4: 85-107.

[40] Varian, Hal. 1980. "A Model of Sales," American Economic Review, 70: 651-659.

[41] Whinston, Michael D. 2006. Lectures on Antitrust Economics (2006).

[42] Wright, Josh. 2008. "Are Loyalty Discounts Really Anticompetitive?" Available at http://truthonthemarket.com/2008/06/11/are-loyalty-discounts-really-anticompetitive/

[43] Wright, Julian. 2008. "Naked Exclusion and the Anticompetitive Accomodation of Entry," Economics Letters, 98:107-112.

Figure 1

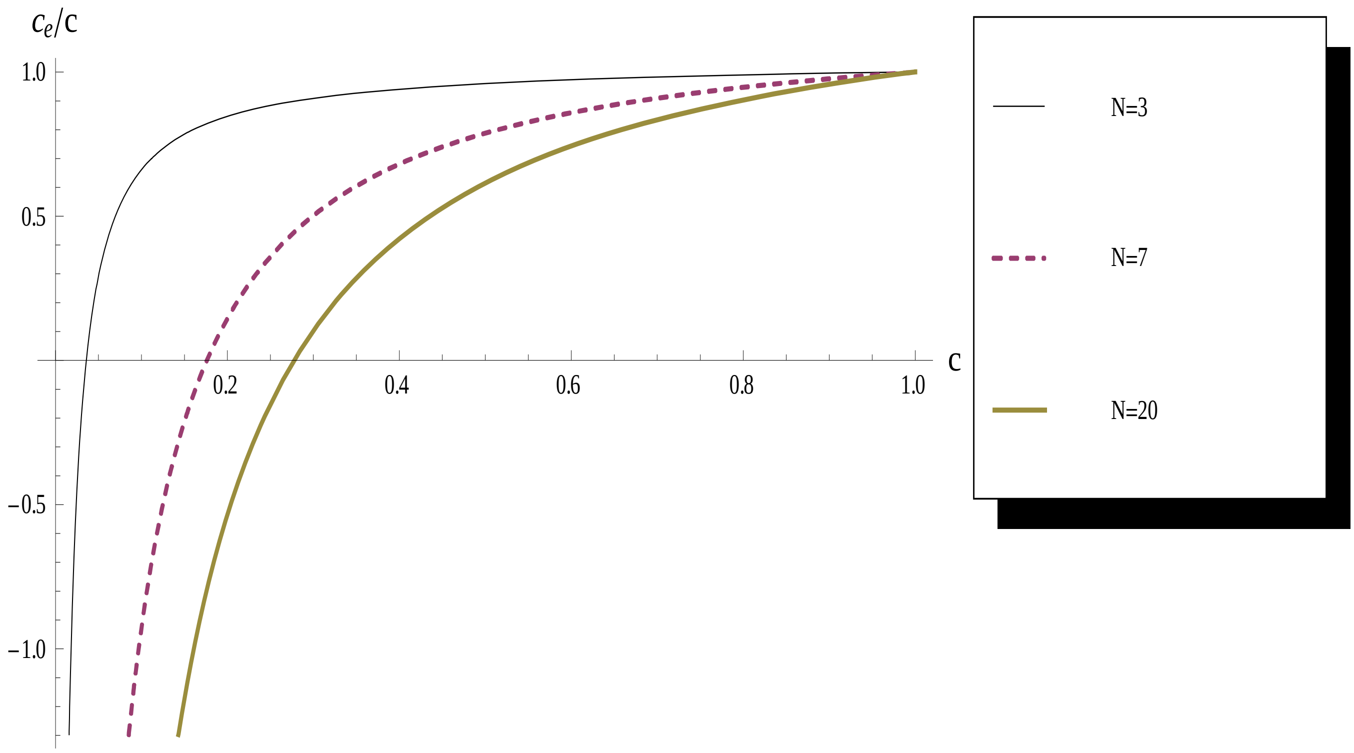
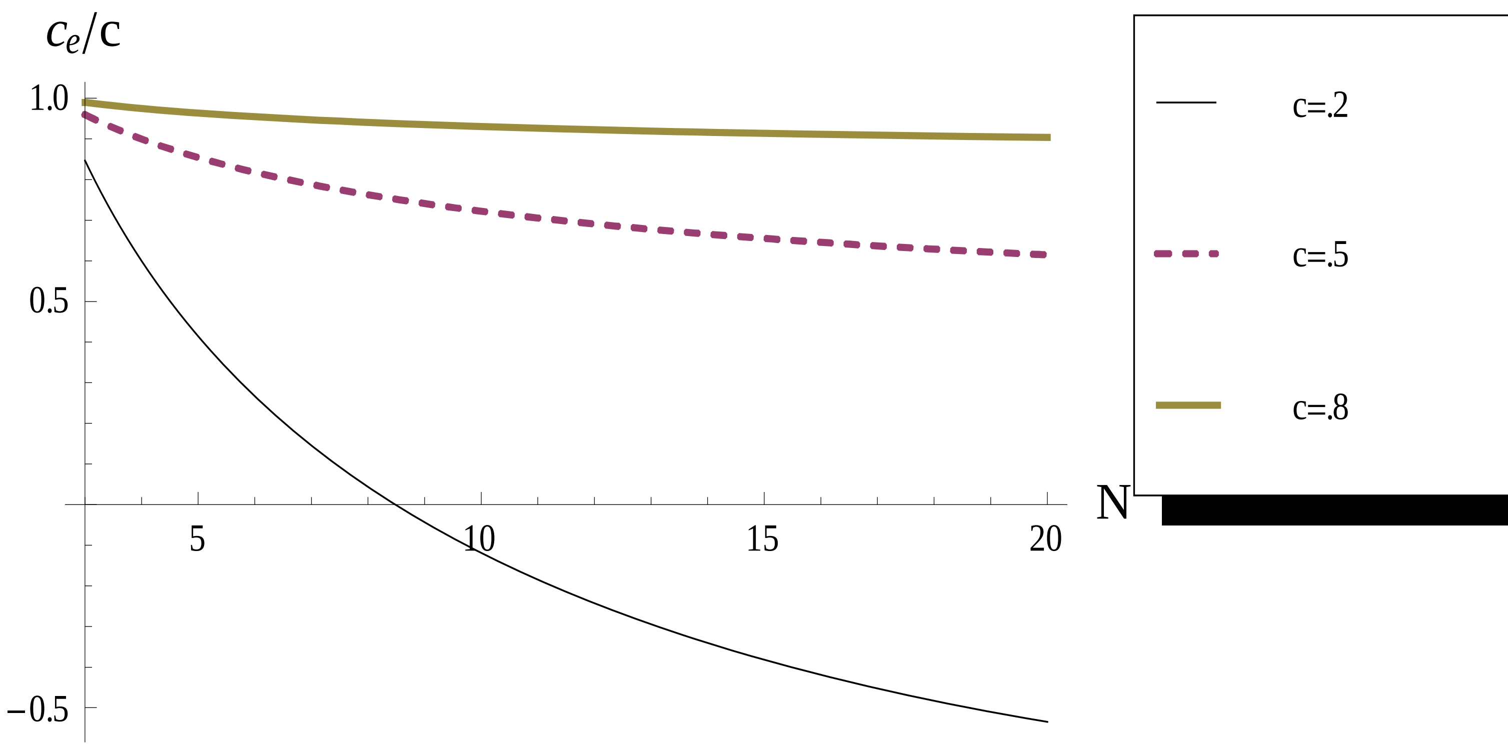



Figure 2



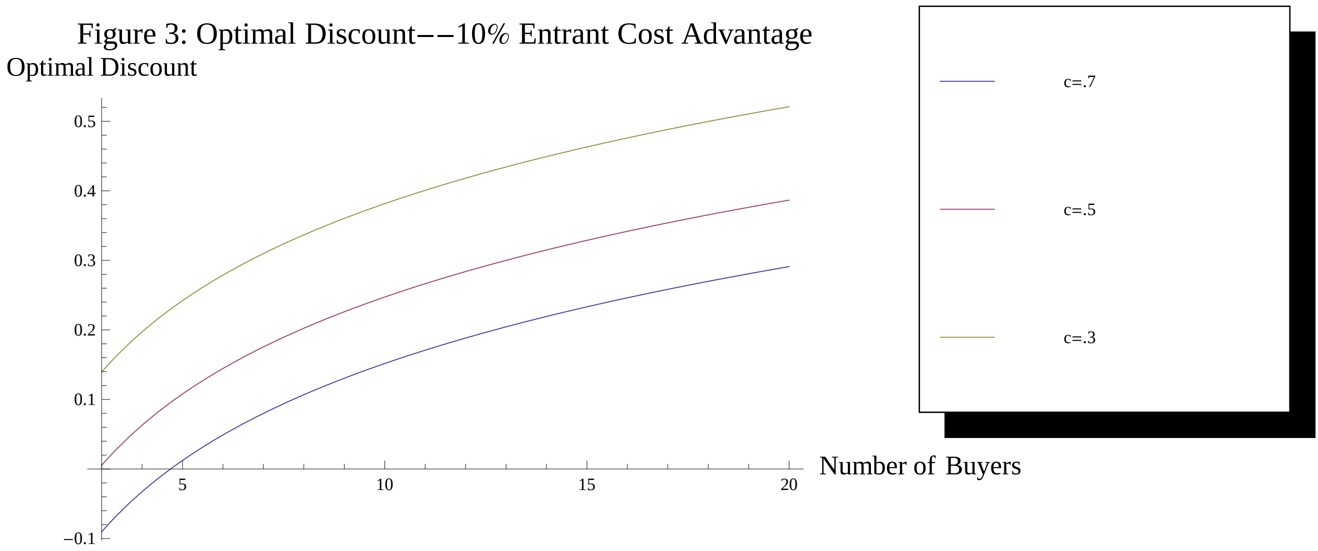


Figure 3: Optimal Discount−−10% Entrant Cost Advantage
Optimal Discount
0.5
0.4
0.3
0.2
0.1
−0.1
5
10
15
20
Number of Buyers
c=.7
c=.5
c=.3